共頁

考試科目	計算机數學(一)	所別	資訊科學	考試時間	4 月 21 日上下午第 節 星期日

第一單元:離散數學部分 (60 %)

I 選擇與填充 (35 %; 不倒扣)

1. (4%) Let f(n) be an increasing function satisfying the recurrence relation: $f(n) = 7\ f(n/2) + 15n^2/4$. Then which of the following statements are correct? (多選)
   (a) $f(n) = O(n^2)$ (b) $f(n) = o(n^2)$ (c) $f(n) = O(n^3)$
   (d) $f(n) = o(n^3)$ (e) $f(n) = \Theta(n^3)$

2. (4%) Which of the following statement *is not* valid in Boolean algebras.
   (a) x(y+z) = xy + xz (b) x(x+y) = x
   (b) (x+y)(x+z) = x + yz (d) x(~y)+(~x)y = x+y

3. (4%) What is the proposition a proof of which *does not* imply the validity of the implication p → q ?
   (a) p → false (b) ~p → ~q (c) ~p or q (d) ~q → false (e) ~(p and ~q)

4. (4%) What is the number of non-negative integer solutions to the equation $x + y + z < 18$ with $x \geq 1$, $y \geq 2$ and $z \geq 3$.

5. (4%) What is the probability of wining the 3 out-of 6 prize in playing Taiwan lottery for correctly choosing 3 (but not 4, 5 or 6) numbers out of six integers chosen between 1 and 42, inclusively, by a fair random process.

6. (5%) What is the least positive integer x satisfying the system of congruences : $x \equiv 2 \pmod 5$, $x \equiv 5 \pmod{11}$ and $x \equiv 11 \pmod{17}$.

7. (10%)Which of the following statements are correct? (多選;每答對一小題給 2%)
   (a) All spanning trees of a graph have the same number of edges.
   (b) If G = (V,E) is a multigraph containing no isolated or pending vertices, then the number of edges |E| of G is always less than or equal to the number of vertices |V| of G.
   (c) The chromatic number of every graph is less than or equal to 4.
   (d) There exists a Eular circuit for every complete simple graph K_5 with 5 vertices.
   (e) There exists one and only one path between every two vertices of a tree.

考試科目	計算機數學(一)	所別	資訊科學	考試時間	4 月 21 日上午第 節 星期日

II 計算與證明 (25 %)

8. (10 %) Let G be a directed graph with n vertices, x and y two distinct vertices of G. Show that if there is a path from x to y then there must exist a path of length less than n from x to y.

9. (15%) Let $\leq$ be a preorder on a nonempty set S (i.e., $\leq$ is a reflexive and transitive relation on S). Define a new relation $\equiv$ on S as follows : for all x, y $\in$ S, x $\equiv$ y iff $x \leq y$ and $y \leq x$.
   (a) Show that $\equiv$ is an equivalence relation on S. [6 %]
   Let $\ll$ be a relation on the set $S/\equiv$ of all equivalence classes of S such that for all A and B belonging to $S/\equiv$, $A \ll B$ iff there exist $x \in A$ and $y \in B$ with $x \leq y$.
   (b) Show that the relation $\ll$ is a partial order on $S/\equiv$. [9 %]

考試科目	計算机數學(一)(機率)	所別	資科系	考試時間	4月21日 星期日 下午第 節

第二單元：機率 (本單元共計 40 分)

[14%] 1. Suppose X and Y are independent random variables uniformly distributed over $\{1, 2, \ldots, n\}$.

(a) Find $P(X \geqq Y)$

(b) Find $P(Z=z)$, where $Z=\max\{X, Y\}$.

[12%] 2. Consider a directed graph G with n nodes. Let X_{ij} be a variable defined so that

$$X_{ij} = \left\{ \begin{array}{ll} 0 & \text{if there is no edge between node } i \text{ and node } j \\ 1 & \text{otherwise} \end{array} \right\}$$

Assume that the $\{X_{ij}\}$ are mutually independent Bernoulli random variables with parameter p. The corresponding graph is called a **p-random-graph**. Find the pmf (probability mass function), the expected value, and the variance of the total number of edges X in the graph.

[14%] 3. Jobs arriving to a computing server have been found to require CPU time that can be modeled by an exponential distribution with parameter 1/140 ms^{-1}. The CPU scheduling discipline is quantum-oriented so that a job not completing within a quantum of 100 ms will be routed back to the tail of the queue of waiting jobs.

(a) Find the probability that an arriving job is forced to wait for a second quantum.

(b) Of the 800 jobs coming in during a day, how many are expected to finish within the first quantum?

備考	試題隨卷繳交