考試科目	線性代數	所別	應用數學系	考試時間	4月20日上午第2節 星期六下

1. Prove or disprove: (48%)

(1) If a square matrix A has orthonormal columns, then A has orthonormal rows.

(2) Let A be a $m\times n$ matrix. Then $\operatorname{rank} A^T A = \operatorname{rank} A$.

(3) The matrix $A_{7\times 7} = B_{7\times 5}C_{5\times 7}$ has no inverse.

(4) There exists a linear transformation on R^2 which maps a rectangle to an ellipse.

(5) Every diagonal entry of a positive definite matrix must be positive.

(6) There exist no square matrices A and B such that $AB - BA = I$.

(7) For every real symmetric $n\times n$ matrix A, there is a real constant k such that the matrix $A + kI_n$ is positive definite.

(8) For every $n\times n$ matrix A, there is a constant k such that $A + kI_n$ is nonsingular.

2. Let $A_{m\times n}x_{n\times 1} = b_{m\times 1}$ be a consistent linear system with real coefficients.

(1) Show that this system has one and only one solution x_0 in $RS(A)$, the row space of A. (Hint: What is the orthogonal complement of $RS(A)$?) (6%)

(2) If x_0 is the solution in $RS(A)$ and x_1 is any other solution of $Ax = b$, show that $\|x_0\| \le \|x_1\|$. The vector x_0 is called the minimal solution of the linear system $Ax = b$. (6%)

(3) Find the minimal solution of $\begin{cases} x+2y+z=4 \\ x-y+2z=-11 \\ x+5y=19 \end{cases}$.

(Hint: Find one solution, then project it into $RS(A)$). (8%)

3. (1) On the surface $-x_1^2 + x_2^2 - x_3^2 + 10x_1x_3 = 1$, find the two points closest to the origin. (6%)

(2) Find the maximum and minimum of $x_1^2 + 2x_2^2 + 3x_3^2 + x_1x_2 + 2x_1x_3 + 3x_2x_3$ on the unit sphere. (6%)

備考	試題隨卷繳交

考試科目	線性代數	所別	應用數學系	考試時間	4 月20日 星期六 下午第2節

4. Let $V = M_{3\times3}(R)$ be the vector space of all 3×3 real matrices. Let

$$A = \begin{bmatrix} 4 & 0 & 1 \\ 2 & 3 & 2 \\ 1 & 0 & 4 \end{bmatrix}.$$

Define a linear operator T on V by $T(B) = AB$ for $B \in V$.

(1) Find the nullspace (kernel) of T.

(2) Evaluate $2A^5 - 11A^4 - 13A^3 + 100A^2 - 50A + +30I$.

(3) Show that the minimal polynomial of T is the same as the minimal polynomial of A.

(4) Is T diagonalizable? Why? (20%)

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