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1. (15 points) In the following case, set up the null hypothesis and the alternative: Explain how you will proceed in testing the hypothesis.

<u>Case</u>: A tire manufacturer advertises that its tires last for at least 30,000 miles. A consumer group does not believe it.

In the problem above, can you identify the costs of mistaken decisions if we view the hypothesis-testing problem as a decision problem?

2. (20 points) In 1970, a random sample of 50 American men aged 35 to 54 showed the following relation between annual income Y (in dollars) and education X (in years).

$$\hat{Y} = 1200 + 800X$$

Average income was  $\overline{Y} = \$10,000$  and the average education was  $\overline{X} = 11.0$  years, with  $\sum X^2 = 900$ . The residual standard deviation about the fitted line was s = \$7300.

- a. Calculate a 95% confidence interval for the population slope.
- b. Is the relation of income to education statistically significant at the 5% significance level?
- c. Predict the income of a man who has completed 2 years of high school (X = 10). Include an interval wide enough that you would bet on it at odds of 95 to 5.
- d. Would it be fair to say that each year's education is worth \$800? Why?
- 3. (25 points) Consider the joint probability density function  $f(x, y) = (0.6)^x (0.4)^{1-x} (0.3)^y (0.52)^{1-y} (2)^{xy}$

where the possible values for X and Y are x = 0.1 and y = 0.1. Find:

- a. The conditional density function f(y | x = 0)
- b. E(X) and Var(X)
- c. Cov(X,Y)
- d. E(X+Y)

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4. (20 points) When S successes occur in n trials, the sample proportion P = S/n customarily is used as an estimator of the probability of success  $\pi$ . However, sometimes there are good reasons to use the estimator  $P^* = (S+1)/(n+2)$ . Alternatively,  $P^*$  can be written as a linear combination of the familiar estimator P:

$$P^* = \frac{nP+1}{n+2} = (\frac{n}{n+2})P + (\frac{1}{n+2})$$

- What is the mean squared error (MSE) of P? Is it consistent?
- What is the mean squared error (MSE) of  $P^*$ ? Is it consistent?
- To decide which estimator is better, P or  $P^*$ , does consistency help? criterion would help?
- Which estimator is better, P or  $P^*$ , when  $n = 10, \pi = 0$ ?
- 5. (20 points) To compare three varieties of potatoes, an experiment was conducted by assigning each variety at random to 3 equal-size plots at each of 3 different soil types. The following yields, in bushels per plot, were recorded:

Soil	Variety of Potato		
	A	В	<u>C</u>
Sand	21	20	16
Clay	16	18	11
Loam	23	31	24

- Construct the ANOVA table
- Calculate the family of 95% simultaneous confidence intervals for the differences in the 3 varieties.
- The botanist who developed variety B remarked that he had worked 10 years to find something that grew well in a loam soil. As you glance at the data, do you think he succeeded? In the light of this information, what would you say about your analysis in parts a and b?

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