考試科目	計算機數學(二)	所別	資科所(甲組) 931	考試時間	4 月 20 日上/下午第 1 節 星期日

離散數學部分 (60 %)

I 選擇與填充 (34 %; 不倒扣)

1. (4%) How many zeros are there at the end of the decimal representation of 400! ?

   (a) 80 (b) 96 (c) 99 (d) 100

2. (4%)How many Boolean functions are there with 4 inputs ?

   (a) 2^4 (b) 2^8 (c) 2^{16} (d) 2^{32}

3. (4%) Which of the following well-formed formulas is not valid under the usual arithmetic interpretation?

   (a) $1\neq 1 \rightarrow 2=3$ (b) $(1=1 \rightarrow 2=2) \leftrightarrow (2\neq 2 \rightarrow 1\neq 1)$

   (a) $(\exists x \forall y\ p(x,y)) \rightarrow \forall y \exists x\ p(x,y)$ (d) $(\forall x \exists y\ p(x,y)) \rightarrow \exists y \forall x\ p(x,y)$

4. (4%) Which of the following languages is *not* regular? Where A and B are two arbitrary regular languages,

   (a) $\{ a^n b^n \mid n \geq 0 \}$ (b) A - B (c) $\{ x \mid \exists y$ with $xy \in A$ and $|x|=|y| \}$

   (d) $\{ a^{f(n)} \mid f(n) = n^2 + n + 1$ and $n \geq 0 \}$

5. (4%) Consider the program S = {x = x+y; y= (y > 0 ? x : -x) ;}. Suppose after the execution of the program the postcondition " x > y " holds, then which of the following conditions must be true before the execution of S ?

   (a) $y < 0$ (b) $x > 0$ (c) $x + y \geq 0$ (d) $x = y$

6. (4%) Suppose f is an increasing function satisfying the divide-and-conquer relation f(n) = 3 f(n/2) + 2n and the initial condition f(1) = 0. What is the asymptotic order of f(n) ?

   (a) $\Theta(n^3)$ (b) $\Theta(n^3 \lg n)$ (c) $\Theta(n^{\lg 3})$ (d) $\Theta(n^2)$

7. (10 %) Suppose that a full 4-ary tree has 27 internal vertices.

   (a) How many leaves does it have ? [4%]

   (b) What is the smallest height it could possibly have ? [4%]

   (c) What is the largest height it could possibly have ? [2%]

   Note: Single-vertex tree is defined to have height 0.

備考	試題隨卷繳交

命題委員： (簽章) 92 年 4 月 /

考試科目	計算機數學(二)	所別	資科所	考試時間	4月20日 星期日 下午第1節

離散數學部分

II 計算與證明 (26 %)

8. (6%) Solve the recurrence relation $a_n = 4\,a_{n-1} + 3\,a_{n-2}$ with the initial conditions $a_0 = 3$ and $a_1 = 13$.
9. (10%) We call a positive integer *perfect* if it equals the sum of all its positive devisors other than itself.
   (a) Find a perfect number in the range (20,30). [3%]
   (b) Show that if $2^p - 1$ is prime, then $2^{p-1}(2^p - 1)$ is perfect. [7%]
10. (10%) Let A be an infinite set and N the set of all non-negative integers. Show that if there is an onto mapping from N to A, then there must exist a 1-1 and onto mapping (bijection) from A to N.

線性代數部分(40%)

(6) 1. If $|A| = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = 2$, find the determinants of the following matrices:

$$B = \begin{bmatrix} 2a_2 & a_1 & 3a_3 \\ 2b_2 & b_1 & 3b_3 \\ 2c_2 & c_1 & 3c_3 \end{bmatrix} \quad \text{and} \quad C = \begin{bmatrix} b_1 & b_2 & b_3 \\ a_1+b_1-c_1 & a_2+b_2-c_2 & a_3+b_3-c_3 \\ 2c_1 & 2c_2 & 2c_3 \end{bmatrix}.$$

(10) 2. Find all values of a for which the resulting linear system has (A) no solution, (B) a unique solution, and (C) infinitely many solutions:

$$\begin{cases} x + y - z = 2 \\ x + 2y + z = 3 \\ x + y + (a^2-5)z = a \end{cases}$$

(9) 3. Find the cosine of the angle between each pair of vectors $\underline{u}$ and $\underline{v}$.

(a) $\underline{u} = (2,3,1)$, $\underline{v} = (3,-2,0)$. (b) $\underline{u} = (2,0,1)$, $\underline{v} = (2,2,-1)$.

(c) $\underline{u} = (0,4,2,3)$, $\underline{v} = (0,-1,2,0)$.

(9) 4. Which of the following are linear transformations? (You must verify or explain your answer

(a) $L(x,y,z) = (x-y,\ x^2,\ 2z)$.

(b) $L\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} 2x-3y \\ 3y-2z \\ 2z \end{bmatrix}$.

(c) $L(x,y) = (x-y,\ 2x+2)$.

(6) 5. Let $A = \begin{bmatrix} 3 & -5 \\ 1 & -3 \end{bmatrix}$. Compute A^{16}.