

考試科目	計算機數學(一)	所別	資科所	考試時間	4月20日上午第1節 星期日(下)
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離散數學部分 (60 %)

I 選擇與填充 (34 %; 不倒扣)

- (4%) How many zeros are there at the end of the decimal representation of $400!$?
(a) 80 (b) 96 (c) 99 (d) 100
- (4%) How many Boolean functions are there with 4 inputs ?
(a) 2^4 (b) 2^8 (c) 2^{16} (d) 2^{32}
- (4%) Which of the following well-formed formulas is not valid under the usual arithmetic interpretation?
(a) $1 \neq 1 \rightarrow 2=3$ (b) $(1=1 \rightarrow 2=2) \leftrightarrow (2 \neq 2 \rightarrow 1 \neq 1)$
(c) $(\exists x \forall y p(x,y)) \rightarrow \forall y \exists x p(x,y)$ (d) $(\forall x \exists y p(x,y)) \rightarrow \exists y \forall x p(x,y)$
- (4%) Which of the following languages is *not* regular? Where A and B are two arbitrary regular languages,
(a) $\{a^n b^n \mid n \geq 0\}$ (b) $A - B$ (c) $\{x \mid \exists y \text{ with } xy \in A \text{ and } |x|=|y|\}$
(d) $\{a^{f(n)} \mid f(n) = n^2 + n + 1 \text{ and } n \geq 0\}$
- (4%) Consider the program $S = \{x = x+y; \quad y = (y > 0 ? x : -x) ;\}$. Suppose after the execution of the program the postcondition " $x > y$ " holds, then which of the following conditions must be true before the execution of S ?
(a) $y < 0$ (b) $x > 0$ (c) $x + y \geq 0$ (d) $x = y$
- (4%) Suppose f is an increasing function satisfying the divide-and-conquer relation $f(n) = 3f(n/2) + 2n$ and the initial condition $f(1) = 0$. What is the asymptotic order of $f(n)$?
(a) $\Theta(n^3)$ (b) $\Theta(n^3 \lg n)$ (c) $\Theta(n^{\lg 3})$ (d) $\Theta(n^2)$
- (10 %) Suppose that a full 4-ary tree has 27 internal vertices.
(a) How many leaves does it have ? [4%]
(b) What is the smallest height it could possibly have ? [4%]
(c) What is the largest height it could possibly have ? [2%]
Note: Single-vertex tree is defined to have height 0.

考試科目	計算機數學(-)	所別	資科所	考試時間	4月20日 上午第1節 星期日 (下)
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離散數學部分

II 計算與證明 (26 %)

8. (6%) Solve the recurrence relation $a_n = 4a_{n-1} + 3a_{n-2}$ with the initial conditions $a_0 = 3$ and $a_1 = 13$.
9. (10%) We call a positive integer *perfect* if it equals the sum of all its positive divisors other than itself.
 - (a) Find a perfect number in the range (20,30). [3%]
 - (b) Show that if $2^p - 1$ is prime, then $2^{p-1}(2^p - 1)$ is perfect. [7%]
10. (10%) Let A be an infinite set and N the set of all non-negative integers. Show that if there is an onto mapping from N to A , then there must exist a 1-1 and onto mapping (bijection) from A to N .

考試科目	計算機數學(一) 機率部份	所別	資訊科學	考試時間	4月20日 星期日	上午第 節
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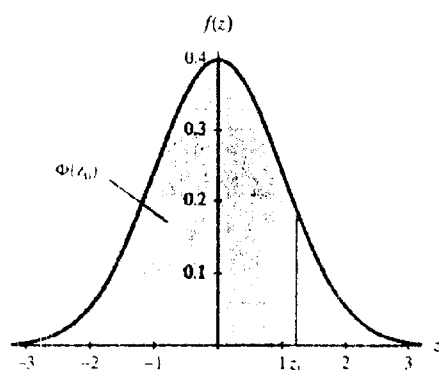
- (9) 1. A die is loaded in such a way that an even number is twice as likely to occur as an odd number. If this die is rolled once, what is the probability that the outcome will be 1 or 2?
- (9) 2. A random variable X has a mean $\mu=10$ and a variance $\sigma^2=16$. Using Chebyshev's theorem, find
- $P(|X-10| \geq 8)$;
 - $P(0 < X < 20)$;
 - the value of the constant c such that $P(|X-10| \geq c) \leq 0.01$.
- (12) 3. A soft-drink machine is regulated so that it discharges an average of 100 milliliters per cup. If the amount of drink is normally distributed with a standard deviation equal to 10 milliliters,
- what fraction of the cups will contain more than 113.5 milliliters?
 - what is the probability that a cup contains between 93.5 and 120.5 milliliters?
 - how many cups will probably overflow if 125 milliliters cups are used for the next 10,000 drinks?
 - below what value do we get the smallest 10% of the drinks?
- (10) 4. A bank has an average of 12 customers between 9:00 am to 12:00 noon. Customers arrive according to a Poisson distribution.
- What is the probability that the time between consecutive arrivals (customers) will fall between six and fifteen minutes?
 - What is the expected time between the arrivals of the next two customers (in minutes)?

考試科目	計算機數學(一)	所別	資訊科學 931	考試時·間	4月20日上午第1節 星期 日 (下)
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計算機數學(一)統計表

656 App. C Tables

Table Va The Normal Distribution



$$P(Z \leq z) = \Phi(z) = \int_{-\infty}^z \frac{1}{\sqrt{2\pi}} e^{-u^2/2} du$$

$$\Phi(-z) = 1 - \Phi(z)$$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7703	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
α	0.400	0.300	0.200	0.100	0.050	0.025	0.020	0.010	0.005	0.001
z_α	0.253	0.524	0.842	1.282	1.645	1.960	2.054	2.326	2.576	3.090
z_{α/2}	0.842	1.036	1.282	1.645	1.960	2.240	2.326	2.576	2.807	3.291