

考試科目	統計學	所別	財政	考試時間	3月20日 上午 8:00 至 11:00 第4節
------	-----	----	----	------	---------------------------

1. (10%) Use the sample $X_1 = 4.9$, $X_2 = 6.5$, $X_3 = 11.5$, and $X_4 = 9.1$ to calculate the maximum likelihood estimator (MLE) for θ in the exponential probability density function (pdf)

$$f_X(x; \theta) = \theta e^{-\theta x}, \quad x \geq 0.$$

2. (10%) Suppose X and Y have a joint probability density function (pdf) given by

$$f_{X,Y}(x, y) = 6x, \quad 0 < x < 1, \quad 0 < y < 1 - x.$$

Find $f_X(x)$ and $f_Y(y)$. Also, graph these two marginal functions.

3. (15%) A random sample of 100 observations from a population produced the following data:

$$\sum_{i=1}^{100} x_i = 1,600 \quad \sum_{i=1}^{100} (x_i - \bar{x})^2 = 6,336.$$

- Calculate the sample mean and sample standard deviation.
- Construct a 95% confidence interval for the population mean using the standard normal distribution.
- Test $H_0: \mu = 19$ against $H_1: \mu \neq 19$ using $\alpha = 0.05$.

4. (15%) The manager of a bank is examining the mortgage payments made by customers of the bank. A payment is classified as good if it arrives on time and as delinquent if it arrives late or is not paid. In addition, the customer's income is classified as low, medium, or high. The following data shows the number of good and delinquent payments according to the person's income category.

Payment	Income level			Total
	Low	Medium	High	
Good	45	50	65	160
Delinquent	5	20	15	40
Total	50	70	80	200

Use a 5% level of significance and test whether the probability of a payment being good or delinquent is independent of the person's income.

$$(\chi_{0.05,2}^2 = 5.99, \chi_{0.025,2}^2 = 7.38, \chi_{0.05,6}^2 = 12.59, \chi_{0.025,6}^2 = 14.45)$$

備 考 試 題 隨 卷 繳 交

命題委員：

104

(簽章)

94年 3 月 7 日

考試科目	統計學	所別	財政	考試時間	3月20日上午第4節 星期日
------	-----	----	----	------	-------------------

5. (15%) A sample of size 1 is drawn from the uniform probability density function defined over the interval $[0, \theta]$.

a. Show that $\hat{\theta} = Y^2$ is not an unbiased estimator for θ^2 .

b. Find an unbiased estimator for θ^2 .

6. (15%) In a sample of 400 products produced by machine 1, 23 were defective, and in a sample of 400 products produced by machine 2, 17 were defective. Test $H_0 = p_1 - p_2 = 0$ against $H_a = p_1 - p_2 \neq 0$ using a 5% level of significance.

7. (20%) You are given the following data on sales (in tens of thousands of NT dollars) and advertising expenditures (in thousands of NT dollars):

Sales, Y	31	40	25	30	20	26
Advertising expenditures, X	5	11	3	4	3	5

a. Calculate the regression equation of sales on advertising expenditures.

b. Find 95% confidence intervals for β_0 and β_1 .

c. Use a 5% level of significance and test $H_0: \beta_1 = 0$ against $H_1: \beta_1 \neq 0$.

d. Predict sales if NT\$6,000 is spent on advertising.

e. Calculate a 95% confidence interval for the mean value of Y if $X = 6$.

($t_{0.025, 4} = 2.776$, $t_{0.025, 5} = 2.571$, $t_{0.025, 6} = 2.447$)

備 考 試 題 隨 卷 繳 交

命 題 委 員 :

105

(簽章)

94年 3 月 7 日