考 試 科 目	微 積 分	所 別	財政學系	考 試 時 間	3 月 20 日 上午 星期 日 下午第 4 節

1. Find the number a such that $\lim\limits_{x \to 0} \dfrac{\sqrt[3]{1+ax}-1}{x}=1$.　　**(5 points)**

2. (1). What is the **Intermediate Value Theorem**? Illustrate it by figures.　　**(10 points)**

   (2). A **fixed point** of a function f is a number c in its domain such that $f(c)=c$. Use the intermediate value theorem to prove that any continuous function with domain $[0, 1]$ and range in $[0, 1]$ must have a fixed point.

3. Suppose f is a function that satisfies the equation　　**(10 points)**
   $f(x+y)=f(x)+f(y)+x^2y+xy^2$ for all real numbers x and y.

   Suppose also that $\lim\limits_{x \to 0} \dfrac{f(x)}{x}=1$. Find

   (1). $f(0)$.　　(2) $f'(0)$.　　(3). $f'(x)$

4. Find the derivative of the following functions.　　**(10 points)**

   (1). $g(x)=\displaystyle\int_{2x}^{3x} \dfrac{u^2-1}{u^2+1}du$.　　(2). $g(x)=\displaystyle\int_{2}^{3} \dfrac{u^2-1}{u^2+1}du$.

5. Evaluate the integral $\displaystyle\int_{1}^{e} \dfrac{x^2+x+1}{x}dx$.　　**(5 points)**

6. If f is continuous on $[a, b]$, show that　　**(5 points)**
   $2\displaystyle\int_{a}^{b} f(x)f'(x)dx=[f(b)]^2-[f(a)]^2$.

7. Evaluate $\lim\limits_{x \to \infty}\left(\dfrac{1}{\sqrt{n}\sqrt{n+1}}+\dfrac{1}{\sqrt{n}\sqrt{n+2}}+\cdots+\dfrac{1}{\sqrt{n}\sqrt{n+n}}\right)$　　**(5 points)**

8. Find the number a such the line $y=a$ divides the region bounded by the curves $y=x^2$ and $y=4$ into two regions with equal area.　　**(5 points)**

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9. (1) Use **mathematical induction** to prove that for $x \geq 0$ and any **(10 points)**

   positive integer n, $e^x \geq 1 + x + \dfrac{x^2}{2!} + \cdots + \dfrac{x^n}{n!}$.

   (2) Use part (1) to show that $\lim\limits_{x \to \infty} \dfrac{e^x}{x^k}$ is divergent for any positive

   integer k.

10. For what values of a does the equation $\ln x = ax^2$ have exactly **(5 points)**
    one solution?

11. A publisher pays the author of a book a royalty of 15%. **(10 points)**
    Demand for the book is $x = 200 - 5p$ and the production cost is
    $C = 10 + x + x^2$.
    (1). Find the optimal sells from both the author's and the
        publisher's perspective.
    (2). Explain why the publisher will always set a higher price and
        sell fewer copies of the book than the author like.

12. A monopolist supplies two markets, one at home, the other **(10 points)**
    abroad. The demand function are $q_h = 10 - p_h$, $q_f = 5 - 0.5p_f$
    where q_h denotes home sale and q_f foreign sale. The firm's total
    cost function is $C = 0.5(q_h + q_f)^2$.
    (1). Find the firm's profit-maximization output and prices.
        (Assume that no arbitrage between the markets is possible).
    (2). Suppose now that price regulation is imposed in the home
        market, in the form of a maximum price of \$6. What is the
        effect of this on prices, outputs, and profit? Illustrate and
        explain your results.

13. Use the demand and supply functions to find the price p as a **(10 points)**
    function of time, t. Begin by setting $D(t)$ equal to $S(t)$ and
    solving the resulting differential equation. Find the general
    solution, and then use the initial condition to find the particular
    solution.

   $D(t) = 4000 + 5p(t) - 4p'(t)$ ...... Demand function

   $S(t) = 2800 + 7p(t) + 2p'(t)$ ...... Supply function

   $p(0) = \$1000.00$ ...... Initial condition

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命 題 委 員 ： 　　　　103　　　　（簽章） 94 年 3 月 7 日