考試科目	線性代數	所別	應用數學 8111 8116	考試時間	3月18日 星期六	第二節

Problem. 1. (15 pts) A matrix M is called skew-symmetric if $M^t = -M$. Denote $M_{n\times n}(\mathbb{C})$ as the set of all $n \times n$ matrices. Let $W_1 \subset M_{n\times n}(\mathbb{C})$ be the set of all $n \times n$ skew-symmetric matrices, and $W_2 \subset M_{n\times n}(\mathbb{C})$ be the set of all $n \times n$ symmetric matrices. Show that $M_{n\times n}(\mathbb{C}) = W_1 \oplus W_2$.

Problem. 2. (15 pts) Let $\mathbb{P}_n(\mathbb{C})$ be the vector space of complex polynomials $f(x)$ of degree at most n, where n is a positive integer. Let D be the matrix of the linear differential operator $\frac{d}{dx}$ acting on $\mathbb{P}_n(\mathbb{C})$ with respect to some basis of $\mathbb{P}_n(\mathbb{C})$. Prove that D is not diagonalizable.

Problem. 3. (15 pts) Prove or give a counterexample: Let A be an $n \times n$ matrix with entries in $\mathbb{R}$, then the eigenvalues of A and A^t are the same.

Problem. 4. (15 pts) Let A be a matrix in $SL(2, \mathbb{C})$. That means A is a 2×2 matrix with complex entries and determinant 1. Suppose the eigenvalues of A are λ_1 and λ_2 (not necessary distinct). Please find all possible pairs of (λ_1, λ_2). No credit will be given if you just write down the answers without proof.

Problem. 5. (15 pts) Let

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 33 & -16 \\ 0 & 48 & -23 \end{pmatrix}.$$

Diagonalize the matrix A or find the Jordan canonical form of A. Suppose D is the result matrix. Find the 3×3 matrix Q and its inverse Q^{-1} such that $Q^{-1}AQ = D$.

Problem. 6. (10 pts) Let $T : \mathbb{R}^n \to \mathbb{R}^n$ be a linear operator. We say a vector $\mathbf{v} \in \mathbb{R}^n$ is a fixed point of T if $T(\mathbf{v}) = \mathbf{v}$. Find the criteria of the eigenvalues of T such that T has a unique fixed point. Justify your answer.

Problem. 7. (15 pts) Let $V = \mathbb{P}(\mathbb{R})$ the the vector space of all real polynomials, with the inner product $\langle f, g\rangle = \int_{-1}^{1} f(t)g(t)\,dt$.

(a) (5 pts) Let $\mathbb{P}_1(\mathbb{R})$ be the subspace of V that contains all real polynomials of degree at most one. Use the Gram-Schmidt Process to find an orthonormal basis for $\mathbb{P}_1(\mathbb{R})$ from the standard basis $\{1, x\}$ of $\mathbb{P}_1(\mathbb{R})$.

(b) (10 pts) Suppose $\mathbf{v}$ is the projection of the vector $4 + 3x - 2x^2$ on $\mathbb{P}_1(\mathbb{R})$. Find the coordinates of the vector $\mathbf{v}$ with respect to the orthonormal basis of $\mathbb{P}_1(\mathbb{R})$ that you found in part (a).

備考	試題隨卷繳交

命題委員： 58 （簽章） 年 月 日

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