國立中山大學100學年度碩士班招生考試試題

科目：微積分丙【企管系甲班碩士班丙組選考】

答案請按題號順序寫在答案卷上，違者扣分。

Are the following statements (1~10) true or false? Give an explanation for your answer or a counterexample if the answer is false.

1. There is a function which is continuous on [0,4] but not differentiable at x=3.(5%)
2. If a function is continuous, then it is differentiable.(5%)
3. If f(a) ≠g(b), then f'(a) ≠ g'(b).(5%)
4. The derivative of a polynomial is always a polynomial.(5%)
5. Suppose f'' and g'' exist and f and g are concave up for all x. Then f(x)g(x) is concave up. (5%)
6. If f'(x)= g'(x), then f(x)= g(x).(5%)
7. If $f''(x)>0$, then $f'(x)$ is increasing.(5%)
8. A critical point of f must be a local maximum or minimum of f.(5%)
9. Since f(x)=1/x is continous for all x>0, and the interval (0,1) is bounded, f has a maximum on the interval (0,1).(5%)
10. If g'(a) ≠0, then $\lim_{x\to a} f(x)/g(x)=f'(x)/g'(x)$.(5%)

Calculate the following (11~18)(5% each)

11. $\int (3e^x + 2\sin x)\,dx$

12. $\int \frac{5}{\sqrt{x}}\,dx$

13. $\int \left(\frac{x+1}{x}\right)dx$

14. $\int (x^4 e^{3x})\,dx$

15. $\int (x^2 e^{x^3+1})\,dx$

16. $\frac{d}{dx}\int_2^{x^3} \sin t^2\,dt$

17. $\int_0^1 \int_y^1 e^{x^2}\,dx\,dy$

18. Approximate $\int_0^1 \sqrt{4-x^2}\,dx$
(hint: using Taylor series)

19. You run a small furniture business. You sign a deal with a customer to deliver up to 400 tables, the exact number to be determined by the customer later. The price will be $900 per table up to 300 tables, and above 300, the price will be reduced by $2.5 per table(on the whole order) for every additional chair over 300 ordered. What are the largest and smallest revenues your company can make under this deal?(10%)