1. (15%) Given any two real-valued random variables X_1 and X_2 with finite second moment. Here, $\mathrm{E}\{\cdot\}$ takes the expectation with respect to X_1 and X_2. If the statement is true, please write a circle ("o") on your ANSWER SHEET. If the statement is wrong, then mark it as ("x") on your ANSWER SHEET. You do NOT need to provide any justification.

   (a) (   ). $(\mathrm{E}\{X_1 X_2\})^2 \leq \mathrm{E}\{X_1^2\}\mathrm{E}\{X_2^2\}$;

   (b) (   ). $\mathrm{E}\{c_1 X_1 + c_2 X_2\} \neq c_1\mathrm{E}\{X_1\} + c_2\mathrm{E}\{X_2\}$, where c_1 and c_2 are constant values;

   (c) (   ). $\mathrm{E}\{(X_1 + X_2)^2\} \leq \mathrm{E}\{X_1^2\} + \mathrm{E}\{X_2^2\}$.

2. (15%). Let Y be a binomial distribution with parameters n and p; i.e., the probability distribution function of Y is given by $P(Y = y) = \binom{n}{y}p^y(1 - p)^{n-y}$, $y = 0, 1, 2, \cdots, n$. Please find
   (a) the mean of Y,
   (b) the variance of Y,
   (c) the probability generating function of Y.

3. (10%) The joint probability density function of the random variable (X_1, X_2) is given by

$$f(x_1, x_2) = \begin{cases} c(x_1 + x_2) & 0 < x_1 < x_2 < 1, \\ 0 & \text{otherwise.} \end{cases}$$

   Are X_1 and X_2 stochastically independent? Why?

4. (10%) Let X and Y be independent normal random variables with zero mean and unit variance. Find the value of $\mathrm{E}\{X^2 Y + XY^2 + X^2 Y^2\}$, in which $\mathrm{E}\{\cdot\}$ takes the expectation with respect to X and Y.

5. (20%) Explain the following terms in detail: central limit theorem, negative correlated, Bayes' theorem, Tchebycheff inequality.

6. (10%) Let X and Y be independent normal random variables with zero mean and standard deviation σ . If $X \cos(\omega t) + Y\sin(\omega t) = R \cos(\omega t - \varphi)$. Find the probability density functions of random variables R and φ respectively.

7. (10%) Let X be an exponential random variable with parameter λ . Find the mean and variance of $2X$.

8. (10%) Let Y be a uniform random variable in the range [a, b]. Find the mean and variance of $6Y$.