

考試科目	微積分	所別	財務管理系	考試時間	2月26日(日) 第二節
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1. (10%) Evaluate $\int_0^1 \int_0^1 xye^{x^2+y^2} dy dx$.

2. (20%) Determine whether the following sequence converges or diverges. If a_n converges, please find $\lim_{n \rightarrow \infty} a_n$.

(a) (5%) $a_n = \frac{4n^2+2}{n^2+3n-1}$

(b) (5%) $a_n = \left(1 + \frac{2}{n}\right)^{n/2}$

(c) (10%) $a_1 = 1, a_{n+1} = 1 + \frac{1}{2}a_n$

3. (14%) Please find the sum of the series:

(a) (7%) $\sum_{n=0}^{\infty} \left[\left(\frac{2}{3}\right)^n - \frac{1}{(n+1)(n+2)} \right]$

(b) (7%) $\sum_{n=0}^{\infty} \frac{2^n}{3^{n+1}n!}$

4. (12%) Please evaluate the following integrals:

(a) (6%) $\int_2^{\infty} \frac{\ln x}{x^2} dx$

(b) (6%) $\int_0^2 x(x^2+1)^5 dx$

5. (24%) Please evaluate the following integrals:

(a) (8%) $\int t^5 \ln(t^7) dt$

(b) (8%) $\int \frac{x+1}{(x-3)^2} dx$

(c) (8%) $\int \frac{-1}{x(\ln x)^2} dx$

6. (10%) Define $y = \left(\frac{x+1}{x-1}\right)^3$, please find $\frac{dy}{dx}$.

7. (10%) Define $y = (7x^2 + 3x - 1)^{-3/2}$, please find dy .