國立高雄大學 101 學年度研究所碩士班招生考試試題

科目：數理統計
考試時間：100 分鐘

系所：
統計學研究所(統計組)
本科原始成績：100 分

是否使用計算機：否

1. A probability function denoted by P is a (set) function which assigns to each event A a number denoted by $P(A)$, called the probability of A, and satisfies the following requirements:
   (R1) $P(A) \geq 0$;
   (R2) $P(S) = 1$, where S is the sample space;
   (R3) For every collection of pairwise disjoint events $A_i, i = 1,2, ...$, we have
   $$P(\cup_i A_i) = \sum_i P(A_i).$$
   Using the above requirements (R1)-(R3) to show the following properties:
   (1) $P(\phi) = 0$, where ϕ is the impossible event.【5%】
   (2) If $A_1 \subseteq A_2$, then $P(A_1) \leq P(A_2)$.【5%】
   (3) $P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2)$.【5%】
2. Suppose that $X \sim N(\mu, \sigma^2)$. Prove that $\frac{X-\mu}{\sigma} \sim N(0,1)$.【15%】
3. Suppose that X is a nonnegative continuous random variable. Prove that $P(X \geq t) \leq \frac{E(X)}{t}$ for any $t > 0$.【10%】
4. Suppose that we observe X_i independent, with $N(\mu, \sigma^2)$ for $i = 1,2, \dots, n$. Let $\bar{X} = \frac{\sum_{i=1}^{n} X_i}{n}$.
   (1) Show that $\bar{X}$ is the maximum likelihood estimator of μ.【5%】
   (2) Show that $\bar{X}$ is an unbiased estimator of μ.【5%】
   (3) Show that $\bar{X}$ is an efficient estimator of μ.【5%】
   (4) Show that $\bar{X}$ is a consistent estimator of μ.【5%】
   (5) Show that $\bar{X}$ is a sufficient statistic.【5%】
5. Suppose that we observe X_i independent, with $N(i\theta, 1)$ for $i = 1,2, \dots, n$. Find the most powerful size-α test for testing that $\theta = 2$ against $\theta = 4$.【10%】
6. (1) State the definition of "$\{X_n\}$ converges almost surely to X".【5%】
   (2) State the definition of "$\{X_n\}$ converges in probability to X".【5%】
   (3) State the definition of "$\{X_n\}$ converges in distribution to X".【5%】
   (4) State the Weak Law of Large Numbers.【5%】
   (4) State the Central Limit Theorem.【5%】