

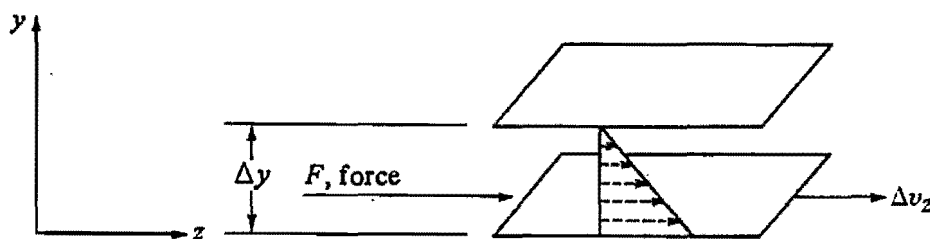


1. Whole milk at 293 K having a density of 1030 kg/m^3 and viscosity of 2.12 cp is flowing at the rate of 0.605 kg/s in a glass pipe having a diameter of 63.5 mm.

- Calculate the Reynolds number. Is this turbulent flow? (10%)
- Calculate the flow rate needed in m^3/s for a Reynolds number of 2100 and velocity in m/s . (10%)

2. Using the below figure, the lower plate is being pulled at a relative velocity of 0.40 m/s greater than the top plate. The fluid used is water at 24°C (viscosity of $0.9142 \times 10^{-3} \text{ Pa} \cdot \text{s}$).

- How far apart should the two plates be placed so that the shear stress τ is 0.30 N/m^2 ? Also, calculate the shear rate. (10%)
- If oil with a viscosity of $2.0 \times 10^{-2} \text{ Pa} \cdot \text{s}$ is used instead at the same plate spacing and velocity as in part (a), what are the shear stress and the shear rate? (10%)





3. A fluid flowing in laminar flow in the x direction between two parallel plates has a velocity profile given by the following

$$v_x = v_{x \max} \left[1 - \left(\frac{y}{y_0} \right)^2 \right]$$

where $2y_0$ is the distance between the plates, y is the distance from the center line, and v_x is the velocity in the x direction at position y . Derive an equation relating $v_{x \text{ av}}$ (bulk or average velocity) to $v_{x \max}$. (10%)



4. Consider a steam pipe of length $L = 30$ m, inner radius $r_1 = 6$ cm, outer radius $r_2 = 10$ cm, and thermal conductivity $k = 20$ W/m. $^{\circ}$ C. The inner and outer surfaces of the pipe are maintained at average temperatures of $T_1 = 180^{\circ}$ C and $T_2 = 60^{\circ}$ C, respectively. (12% all)

(a). Assume: one-dimensional heat conduction in the r direction only, i.e. $T = T(r)$, steady-state, and there is no heat generation. The heat equation can be derived as:

$$\frac{d}{dr} \left(r \frac{dT}{dr} \right) = 0$$

Together with the following boundary conditions:

$$T(r_1) = T_1 \quad \text{and} \quad T(r_2) = T_2$$

Derive that the temperature distribution inside the pipe is:

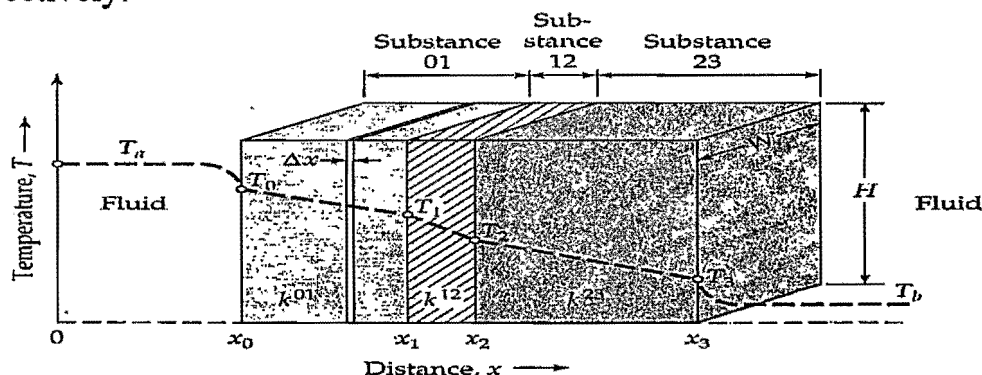
$$T(r) = \frac{(T_2 - T_1)}{\ln(r_2 / r_1)} \ln(r / r_1) + T_1. \quad 4\%$$

(b). **Calculate** the heat flux at $r = 8$ cm. 4%

(c). **Calculate** the rate of heat conduction through the pipe. 4%

5. Heat conduction through composite walls (14% all)

(a). As shown in the following figure, a composite wall is made up of three materials of different thicknesses, $x_1 - x_0$, $x_2 - x_1$, and $x_3 - x_2$, and different conductivities k_{01} , k_{12} , and k_{23} . At $x = x_0$, substance 01 is in contact with a fluid at temperature T_a , and at $x = x_3$, substance 23 is in contact with a fluid at temperature T_b . The convective heat transfer coefficients at the boundaries $x = x_0$ and $x = x_3$ are h_0 and h_3 , respectively.





Assume: one-dimensional heat conduction in the x direction only, i.e. $T = T(x)$, steady-state, and there is no heat generation. **Derive that** the heat flux can be calculated by:

$$q_0 = U(T_a - T_b)$$

where U , called the “overall heat transfer coefficient,” is given by:

$$\frac{1}{U} = \frac{1}{h_0} + \frac{x_1 - x_0}{k_{01}} + \frac{x_2 - x_1}{k_{12}} + \frac{x_3 - x_2}{k_{23}} + \frac{1}{h_3} \quad 5\%$$

Hint:

The energy balance equation $\frac{d}{dx}q_x = 0$ and the Fourier's law $q_x = -k \frac{dT}{dx}$.

(b). Now, consider only two layers. A composite wall is made up of two materials of different thicknesses, $L_1 = 0.004$ m and $L_2 = 0.01$ m, different conductivities $k_1 = 0.78$ W/m.°C and $k_2 = 0.026$ W/m.°C, and same surface areas $A = 1.2$ m². The temperatures of the two fluid streams are $T_a = 30^\circ\text{C}$ and $T_b = 0^\circ\text{C}$. The convective heat transfer coefficients at the boundaries are $h_a = 10$ W/m².°C and $h_b = 40$ W/m².°C. **Calculate:**

(i). The overall heat transfer coefficient U and the heat flux q_0 . 4%

(ii). The temperatures at the boundaries of the two materials T_1 , T_2 , and T_3 . 5%

Note: The temperatures are in the order of $T_a \rightarrow T_1 \rightarrow T_2 \rightarrow T_3 \rightarrow T_b$.



6. Air at 70°F (T_{b1}) and 1 atm is to be pumped through a straight 2-in (D) i.d. tube at a rate of 70 lb_m/hr (w). A section of the tube is to be heated to an inside wall temperature of 250°F (T_o) to raise the air temperature. The heated length is 20 ft (L). The physical properties of air are as follows: viscosity $\mu = 0.05$ lb_m/hr-ft, specific heat $C_p = 0.242$ Btu/lb_m-°F, and thermal conductivity $k = 0.018$ Btu/hr-ft-°F. **Calculate:** (12% all)

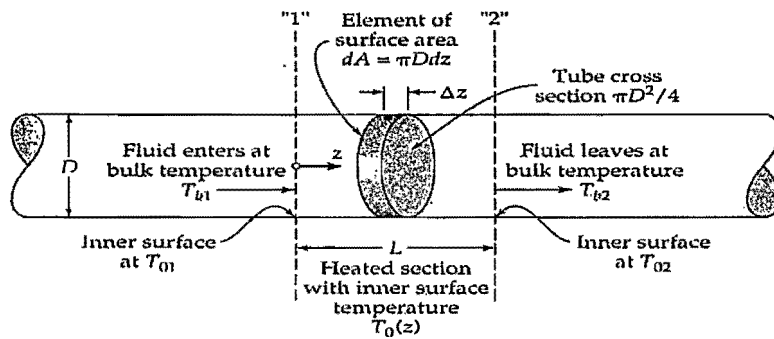
(a). The logarithmic mean heat transfer coefficient h_{lm} . 4%

(b). The bulk temperature of air at the exit of the heated region T_{b2} . 4%

(c). The rate of heat flow Q . 4%

Hint:

As shown in the following figure, a fluid flows through a circular tube of diameter D , in which there is a heated wall section of length L and varying inside surface temperature $T_o(z)$, going from T_{o1} to T_{o2} . The bulk temperature T_b of the fluid increases from T_{b1} to T_{b2} in the heated section.



BIRD: *Transport Phenomena*, 2e
Fig. 14.1-1 W-198

The logarithmic mean temperature difference ΔT_{\ln} and the logarithmic mean heat transfer coefficient h_{\ln} are defined as:

$$Q = h_{\ln} (\pi D L) \Delta T_{\ln}$$

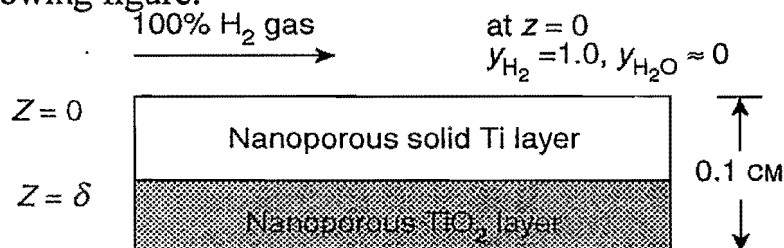
$$\Delta T_{\ln} = (T_0 - T_b)_{\ln} = [(T_{01} - T_{b1}) - (T_{02} - T_{b2})] / [\ln(T_{01} - T_{b1}) - \ln(T_{02} - T_{b2})]$$

where the rate of heat flow Q can also be calculated by:

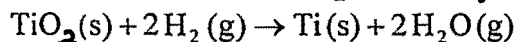
$$Q = w C_p (T_{b2} - T_{b1})$$

h_{\ln} can be estimated by the following correlations: for highly turbulent flow ($Re > 20,000$), $Nu_{\ln} = 0.026 Re^{0.8} Pr^{1/3}$ and for laminar flow, $Nu_{\ln} = 1.86 (Re Pr D / L)^{1/3}$. The dimensionless parameters are defined as: $Pr = C_p \mu / k$, $Re = D v_b \rho / \mu = 4 w / (\pi D \mu)$, and $Nu_{\ln} = h_{\ln} D / k$.

7. As part of the manufacturing process for the fabrication of titanium-oxide-based solar panels, a layer of nonporous titanium oxide must be reduced to metallic titanium, Ti, by hydrogen gas as shown in the following figure: (12% all)



The reaction at the Ti/TiO₂ boundary is given by:



Let species A represent H₂(g) and species B represent H₂O(g). Further assume: (1). The non-homogeneous reaction occurs only at the TiO₂/Ti surface, i.e. there is no reaction occurring when species A (H₂) diffuses in the Ti layer ($R_A=0$); (2) The diffusion of species A (H₂) is under steady state and one-dimensional (z-direction) conditions.



- (a). Using the following boundary conditions: $z = 0$, $y_A = 1.0$ and $z = \delta$, $y_A = 0$, derive that $N_{A,z} = \frac{c D_{AB}}{\delta}$. 6%

Where c , D_{AB} are the gas phase concentration and diffusivity, respectively.

- (b). Further consider a pseudo steady-state condition for the growth of the Ti layer (thickness δ). Using the following boundary conditions: $t = 0$, $\delta = \delta_1$ and $t = \theta$, $\delta = \delta_2$, derive that $\theta = \frac{\rho_{Ti} / M_{Ti}}{c D_{AB}} (\delta_2^2 - \delta_1^2)$. 6%

Where ρ_{Ti} , M_{Ti} are the density and molecular weight of Ti, respectively.

Hint:

The general differential equation for mass transfer of species A :

$$\frac{\partial c_A}{\partial t} + \left[\frac{\partial N_{A,x}}{\partial x} + \frac{\partial N_{A,y}}{\partial y} + \frac{\partial N_{A,z}}{\partial z} \right] = R_A$$

Fick's equation of species A :

$$N_{A,z} = -c D_{AB} \frac{d y_A}{d z} + y_A (N_{A,z} + N_{B,z})$$