國立雲林科技大學
100學年度碩士班暨碩士在職專班招生考試試題

系所：電機系
科目：線性代數(1)

1. Let $A=\begin{bmatrix}1 & 2 & -3 & 4\\ -4 & 2 & 1 & 3\\ 3 & 0 & 0 & -3\\ -1 & -2 & 1 & -1\end{bmatrix}$,

   (a) (8%) Find the determinant of A

   (b) (8%) Compute the rank of A

2. Let the set of vectors $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ be linearly independent. Determine whether the following sets of vectors are linearly dependent or independent.

   (a) (8%) $\{\mathbf{v}_1+\mathbf{v}_2, \mathbf{v}_2+\mathbf{v}_3, \mathbf{v}_3+\mathbf{v}_1\}$

   (b) (8%) $\{\mathbf{v}_1-\mathbf{v}_2, \mathbf{v}_2-\mathbf{v}_3, \mathbf{v}_3-\mathbf{v}_1\}$

3. (18%) Compute the eigenvalues and associated eigenvectors of $A=\begin{bmatrix}0 & 0 & 3\\ 1 & 0 & -1\\ 0 & 1 & 3\end{bmatrix}$.

4. (4%) Write down a 3×3 matrix A so that if the vector $v=(x, y, z)$ in $\mathbf{R}^3$ is multiplied by A, the x and y coordinates of v are unchanged, but the z coordinate becomes zero.

5. (10%) Find a unit vector orthogonal to $u=4\mathbf{i}-6\mathbf{j}+\mathbf{k}$ and $v=2\mathbf{i}+\mathbf{j}-3\mathbf{k}$.

6. Consider $a=(1,-1, 0, 0)$, $b=(0, 1,-1, 0)$, and $c=(0, 0, 1,-1)$.

   (a) (8%) Find the orthonormal vectors A, B, C by Gram-Schmidt operations from a, b, and c.

   (b) (8%) Show that $\{A, B, C\}$ and $\{a, b, c\}$ are bases for the space of vectors perpendicular to $d=(1, 1, 1, 1)$.

7. (8%) Given $A=\begin{bmatrix}1 & 0\\ -2 & 1\\ 1 & 3\end{bmatrix}$ and $b=\begin{bmatrix}2\\ 3\\ 0\end{bmatrix}$, find the projection of b onto the column space of A by solving $A^TA\hat{x}=A^Tb$ and $p=A\hat{x}$.

8. (12%) Find the least squares parabola for the data points $\{(1, 2), (2, 5), (3, 7), (4, 1)\}$.