考生作答須知：
一、本試卷共有兩部分，各佔50分。每題配分標示於題後，總分100分。
二、答案請於答案卷上依題號次序作答，題號務必標示清楚，並寫出計算過程，否則不予計分。

第一部分：微分 (50%)

1. (10%) Find the following limits:

(1) (5%) $\lim_{x\to 0}\dfrac{\sqrt{1+3x}-\sqrt[3]{1+5x}}{x}$　　(2) (5%) $\lim_{x\to 0}\dfrac{1-\cos x}{x^{10}}$

2. (10%) Find the sum of $S=\sum_{k=0}^{\infty}(-1)^k\left(\dfrac{2k+1}{2^k}\right)$.

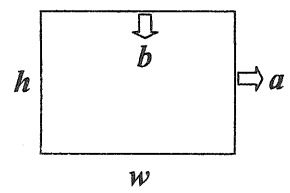
3. (10%) Find the derivatives of the following.

(1) (5%) Let $y=\dfrac{(1-x)^2}{(1+x)^4}$, in which $x\neq -1$. Find the derivative of $\dfrac{dy}{dx}$.

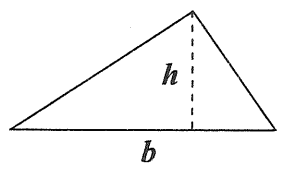
(2) (5%) Let $w=x^2+y^2$, in which $y=y(x)$ is the function defined by

$x^2-xy+y^2-1=0$. Find the derivative of $\dfrac{dw}{dx}$.

4. (10%) Given a rectangle with width $w$ meters and height $h$ meters.
Assume the width is *increasing* by $a$ meters per second
and the height is *decreasing* $b$ meters per second.
(1) (5%) What is the rate of change of the diagonal length?
(2) (5%) What is the rate of change of the rectangle area?




5. (10%) Given a triangle with base $b$ and height $h$.
What is the maximum area of the rectangle
that can be put inside the given triangle?



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第二部分：積分 (50%)

1. Evaluate each integral of the following: (24%)

(a) $\int \frac{x^2}{(x-a)(x-b)(x-c)}dx$.

(b) $\int_0^2 \max\{3x, 4-x^2\}dx$.

(c) $\int_{-\infty}^{\infty} e^{(x-e^x)}dx$.

(d) $\int_0^{\infty} \frac{e^{-ax}-e^{-bx}}{x}dx$, where $a>0$ and $b>0$.

2. Determine the arc length of the graph of $f(x)=\int_0^x \sqrt{t^2+2t}\,dt$ for $2\le x\le 4$. (8%)

3. Determine the volume of the solid obtained by revolving the region bounded by the curve $x^2+(y-b)^2\le a^2$, where $0<a<b$, about the $x$-axis. (10%)

4. Use Simpson's Rule (also called the Parabolic Rule) with $n$=8 subintervals to estimate $\int_{1/2}^{2} \frac{\sin x}{x}dx$ to two decimal places. (8%)