國立中正大學103學年度碩士班招生考試試題

電機工程學系-信號與媒體通訊組

系所別：通訊工程學系-通訊系統組　　科目：通訊原理

通訊工程學系-網路通訊甲組

一、單選題(共 30 分)：每題有五個選項，選擇一個最適當的答案，每題答對得 5 分；未作答、答錯或答多於一個選項者，該題以 0 分計算。

1. Let $m_1(t)$ and $m_2(t)$ be two different real-valued message signals with bandwidth W Hz. Which of the following statements is false ?

   (a) The signal $A_c m_1(t)\cos(2\pi f_c t)$ has bandwidth $2W$.

   (b) The Hilbert transform of $A_c m_1(t)\cos(2\pi f_c t)$ has bandwidth $2W$.

   (c) The signal $A_c m_1(t)\cos(2\pi f_c t) + A_c m_2(t)\sin(2\pi f_c t)$ has bandwidth $2W$.

   (d) The signal $A_c m_1(t)\cos(2\pi f_c t) + A_c \hat{m}_1(t)\sin(2\pi f_c t)$ has bandwidth $2W$, where $\hat{m}_1(t)$ denotes the Hilbert transform of $m_1(t)$.

   (e) The signal $A_c m_1(t)\sin(2\pi f_c t)$ has bandwidth $2W$

2. Let $x(t)$ and $y(t)$ be two aperiodic complex-valued energy-type signals with Fourier transform $X(f)$ and $Y(f)$, respectively. Let $()^*$ denote the complex conjugate operation. Which of the following statements is false ?

   (a) The Parseval's property states that $\int_{-\infty}^{\infty} x(t)y^*(t)dt = \int_{-\infty}^{\infty} X(f)Y^*(f)df$.

   (b) The Fourier transform of $\int_{-\infty}^{\infty} x(\tau)y(t-\tau)d\tau$ is $X(f)Y(f)$.

   (c) The Fourier transform of $x(t)+x^*(t)$ is a real-valued function of f.

   (d) The Fourier transform of $X(t)$ is $x(-f)$.

   (e) The Fourier transform of $x(t)+x(-t)$ is an even function of f.

3. Two Gaussian random variables X and Y have a joint probability density function

$$f(x,y) = \frac{1}{2\pi\sigma^2\sqrt{1-\rho^2}}\exp\left[-\frac{x^2-2\rho xy+y^2}{2\sigma^2\left(1-\rho^2\right)}\right],$$

where $\sigma^2 > 0$ and ρ are two parameters. Which of the following statements

is false ?

(a) The mean of random variable X is 0.

(b) The variance of random variable X is σ^2.

(c) The marginal probability density function of random variable Y is

$$f(y)=\frac{1}{\sqrt{2\pi}\sigma}\exp\left[-\frac{y^2-2\rho y+\rho^2}{2\sigma^2}\right].$$

(d) The correlation of two random variables $E\{XY\}$ is equal to $\rho\sigma^2$.

(e) The random variables X and Y are correlated.

4. Let X_n be an independent identically Gaussian distributed random process (sequence) with mean μ and variance σ^2. Which of the following statements is false ?

(a) It is strictly sense stationary.

(b) It is wide-sense stationary.

(c) The process $Z_n=X_n-X_{n-1}$ is white.

(d) At a particular time instant n, random variable X_n has density function

$$f(x)=\frac{1}{\sqrt{2\pi}\sigma}\exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right].$$

(e) The power spectral density function of X_n is flat.

5. Let $\{\phi_1(t),\phi_2(t),\phi_3(t)\}$ be a set of orthonormal signals over the interval $[0,T_s]$. Suppose we want to design a system to transmit a binary information over an additive white Gaussian noise channel. If the logic 0 is represented by $x(t)=\frac{1}{\sqrt{3}}\phi_1(t)-\frac{1}{\sqrt{3}}\phi_2(t)+\frac{1}{\sqrt{3}}\phi_3(t)$, which of the following signals is the best candidate for logic 1 ?

(a) $-\frac{1}{\sqrt{3}}\phi_1(t)+\frac{1}{\sqrt{3}}\phi_2(t)-\frac{1}{\sqrt{3}}\phi_3(t)$.

(b) $\frac{-1}{\sqrt{3}}\phi_1(t)-\frac{1}{\sqrt{3}}\phi_2(t)+\frac{1}{\sqrt{3}}\phi_3(t)$

(c) $\frac{-1}{\sqrt{3}}\phi_1(t)-\frac{1}{\sqrt{3}}\phi_2(t)-\frac{1}{\sqrt{3}}\phi_3(t)$

(d) $\frac{1}{\sqrt{3}}\phi_1(t)+\frac{1}{\sqrt{3}}\phi_2(t)-\frac{1}{\sqrt{3}}\phi_3(t)$

(e) $\frac{1}{\sqrt{3}}\phi_1(t)-\frac{1}{\sqrt{3}}\phi_2(t)-\frac{1}{\sqrt{3}}\phi_3(t)$

6. Consider a set of binary codewords $\mathbf{c}_1=[1,1,1,1]$, $\mathbf{c}_2=[1,-1,1,-1]$ $\mathbf{c}_3=[1,-1,-1,-1]$, $\mathbf{c}_4=[1,-1,-1,1]$, and $\mathbf{c}_5=[1,1,-1,-1]$. Assume that the received signal is $\mathbf{r}=\alpha\mathbf{c}_m+\mathbf{w}$, where $\alpha=0.5$ is the known channel gain and $\mathbf{w}$ is the additive white Gaussian noise whose entries are of zero mean and variance σ^2. Suppose that $\mathbf{r}=[0.2,0.3,-0.1,-0.2]$ is received, the maximum likelihood detector determines the received codeword to be

(a) $\mathbf{c}_1$

(b) $\mathbf{c}_2$

(c) $\mathbf{c}_3$

(d) $\mathbf{c}_4$

(e) $\mathbf{c}_5$

二、計算題(共40分)：

1. (10分) A real-valued random process $Y(t)=X(t)+X(t-T)$, where $X(t)$ is a wide-sense stationary process with power spectral density $S_X(f)$. Express the power spectral density of $Y(t)$ in terms of $S_X(f)$.

2. (10分) Suppose signal $x(t)$is periodic with period T. Then $x(t)$can be represented by its Fourier series representation $x(t)=\sum_{k=-\infty}^{\infty}X_k e^{j2\pi kt/T}$. Let the Fourier series representation of $y(t)=\hat{x}(t)=\sum_{k=-\infty}^{\infty}Y_k e^{j2\pi kt/T}$, where $\hat{x}(t)$ is the Hilbert transform of $x(t)$. Express the Fourier series coefficients Y_k in terms of

the coefficients X_k.

3. (10 分) Let $x(t) = \sum_{k=-\infty}^{\infty} p(t - nT)$ be a periodic signal with period T and

$$p(t) = \begin{cases} 1, & -\frac{T}{4} < t < \frac{T}{4} \\ 0, & \text{otherwise.} \end{cases}$$

a). Determine the continuous-time Fourier transform $X(f)$ of $x(t)$.

b). Plot $X(f)$ versus frequency f

4. (10 分) Consider the case of binary PAM signals over the AWGN channel in which the two possible signal points are $s_1 = -s_2 = \sqrt{E_b}$, where E_b is the energy per bit . The prior probabilities are $P(s_1) = p$ and $P(s_2) = 1 - p$. The receive signal (one dimension) can be written as $r = \pm\sqrt{E_b} + w$, where w is the AWGN distributed according to $\mathcal{N}(0, \sigma_n^2)$.

a). Determine the metrics for the maximum likelihood (ML) detector when the transmitted signal is corrupted by AWGN.

b). Derive the probability of error for detecting binary PAM signals by the ML detector.

三、名詞解釋(共 30 分)：請以下列兩名詞為標題，利用數學符號、數學式、圖、表格或其他專業術語寫兩篇短文(每篇至多 500 字)，從該名詞的定義、用途、特性等，分別解釋下列的名詞。

1. (15 分) Frequency Division Multiplexing
2. (15 分) Quadrature Amplitude Modulated (QAM) Signals