1. (10%) Compute and plot $y[n]=x[n]*h[n]$, where

$$x[n]=\begin{cases}1, & 2\le n\le 7\\ 0, & \text{otherwise}\end{cases}$$

$$h[n]=\begin{cases}1, & 3\le n\le 13\\ 0, & \text{otherwise}\end{cases}$$

2. (10%) A continuous-time periodic signal $x(t)$ is real valued and has a fundamental period T=16. The nonzero Fourier coefficients for $x(t)$ are

$$a_1=a_{-1}=3,\ a_3=a_{-3}^*=6j.$$

Express $x(t)$ in the form $x(t)=\sum_{k=0}^{\infty}A_k\cos\left(\omega_k t+\phi_k\right)$.

3. (15%) An LTI system with impulse response $h_1[n]=\left(\frac{1}{2}\right)^n u[n]$ is connected in parallel with another causal LTI system with impulse response $h_2[n]$. The resulting parallel interconnection has the frequency response

$$H(e^{j\omega})=\frac{8}{8-3e^{-j\omega}+e^{-j2\omega}}$$

Determine $H_2(e^{j\omega})$ and $h_2[n]$.

4. A stable LTI system is characterized by the differential equation below.

$$\frac{d^2y(t)}{dt^2}+6\frac{dy(t)}{dt}+8y(t)=2x(t)$$

(a) (5%) Determine the frequency response of the system.

(b) (5%) Determine the impulse response of the system.

(c) (5%) What is the response of this system if $x(t)=te^{-2t}u(t)$?

5. (10%) Consider the signal $x(t)$ shown in Figure 1. Determine its Fourier transform.

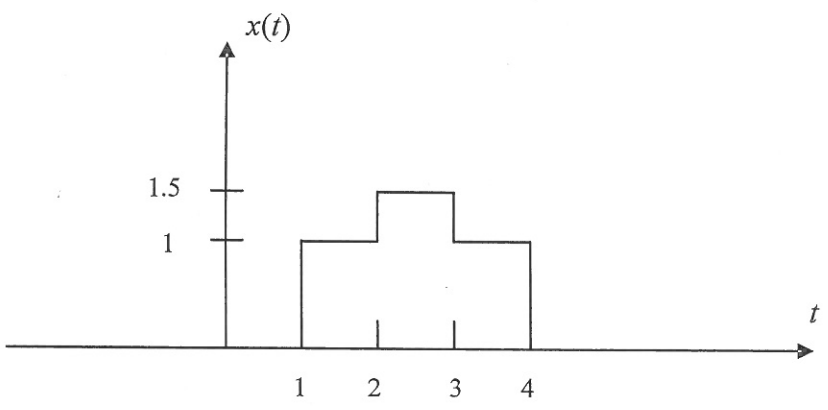

Figure 1.

6. (10%) Consider a discrete-time LTI system with frequency response

$$H(e^{j\omega})=\begin{cases}1, & |\omega|\le\frac{\pi}{4}\\ 0, & \frac{\pi}{4}<|\omega|\le\pi\end{cases}.$$

Show that if the input $x[n]$ to this system has a period N=4, the output y[n] has only one nonzero Fourier series coefficient per period.

7. Consider a discrete-time sequence $x[n]$ from which we form two new sequences, $x_u[n]$ and $x_d[n]$, wher $x_u[n]$ corresponds to sampling $x[n]$ with a sampling period of 2 and $x_d[n]$ corresponds to decimating $x[n]$ by a factor of 2, so that

$$x_u[n]=\begin{cases}x[n], & n=0,\pm2,\pm4,...\\ 0, \text{ otherwise}\end{cases}$$

$$x_d[n]=x[2n].$$

(a) (10%) If $x[n]$ is as illustrated in Figure 2 (a), sketch the sequences $x_u[n]$ and $x_d[n]$.

(b) (10%) If $X(e^{j\omega})$ is as shown in Figure 2(b), sketch $X_u(e^{j\omega})$ and $X_d(e^{j\omega})$.

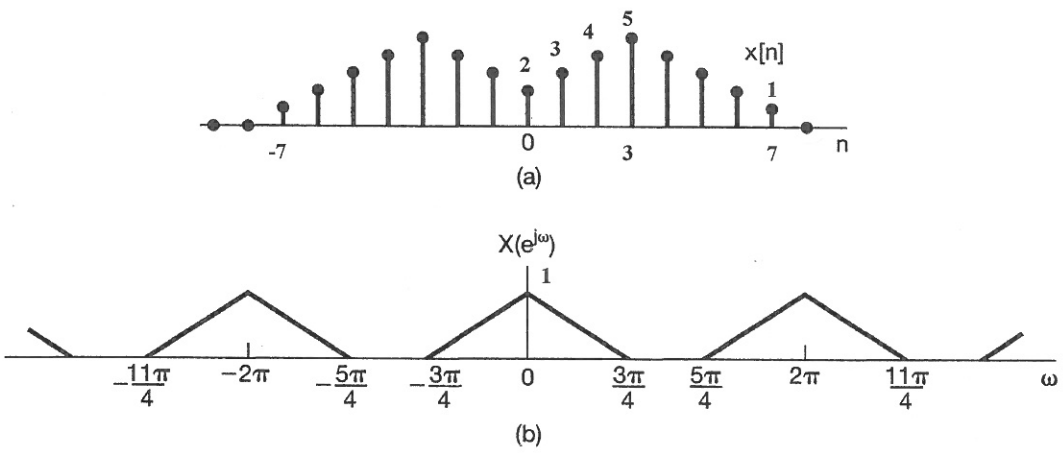

Figure 2.

8. Draw block diagram representations for causal LTI systems described by the following difference/differential equations:

(a) (5%) $y[n]=\frac{1}{3}y[n-1]+\frac{1}{2}x[n-1]$

(b) (5%) $\frac{dy(t)}{dt}+2y(t)=\frac{1}{3}x(t)$