

國立中央大學103學年度碩士班考試入學試題卷

所別：統計研究所碩士班 不分組(一般生) 科目：數理統計 共 2 頁 第 1 頁

統計研究所碩士班 不分組(在職生)

本科考試可使用計算器，廠牌、功能不拘

*請在試卷答案卷(卡)內作答

- Let X_1, X_2 and X_3 denote a random sample from the distribution having p.d.f. $f(x) = e^{-x}, 0 < x < \infty$. Show that

$$Y_1 = \frac{X_1}{X_1 + X_2}, Y_2 = \frac{X_1 + X_2}{X_1 + X_2 + X_3}, Y_3 = X_1 + X_2 + X_3$$
are mutually independent. (15%)
- Let X has the logistic p.d.f. $f(x) = e^{-x} / (1 + e^{-x})^2, -\infty < x < \infty$.
 (a) Find the distribution function $F(x)$ of X . (5%)
 (b) Show that $f(x) = F(x)[1 - F(x)]$. (5%)
 (c) Show the moment-generating function $M(t)$ of X is
 $\Gamma(1-t)\Gamma(1+t), -1 < t < 1. (\Gamma(\alpha) = (\alpha-1)!)$ (5%)
- Let random variables X, Y be jointly distributed with p.d.f. given by

$$f(x, y) = \frac{2}{n(n+1)}$$
where $y = 1, \dots, x; x = 1, \dots, n$.
 Compute
 (a) $E(X|Y = y)$, (5%)
 (b) $E(Y|X = x)$, (5%)
 (c) correlation coefficient $\rho(X, Y)$. (5%)
- Let X_1, X_2 be two random variables with joint m.g.f. given by

$$M(t_1, t_2) = \left[\frac{1}{3}(e^{t_1+t_2} + 1) + \frac{1}{6}(e^{t_1} + e^{t_2}) \right]^2, t_1, t_2 \in R.$$
 Calculate
 (a) $E(X_1)$ (5%)
 (b) $\text{Var}(X_1)$ (5%)
 (c) $\text{Cov}(X_1, X_2)$ (5%)
- Let X_1, \dots, X_n be a random sample from $N(\mu, \sigma^2)$, where both μ and σ^2 are unknown. Find the UMVUE estimator of μ/σ . (10%)

參考用

注意：背面有試題

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6. Let X_1, \dots, X_n be i.i.d. random variables with p.d.f. given by

$$f(x) = \frac{1}{\theta} e^{-x/\theta}, 0 < x < \infty, 0 < \theta < \infty.$$

(a) Derive the UMP test for testing the hypothesis

$$H_0: \theta \geq \theta_0 \text{ vs. } H_1: \theta < \theta_0 \text{ at level of significance } \alpha. \quad (10\%)$$

(b) Determine the minimum sample size n required to obtain power at least 0.95 against the alternative $\theta_1 = 500$ when $\theta_0 = 1000$ and $\alpha = 0.05$. (10%)

7. Let X_1, \dots, X_n be i.i.d. random variables with p.d.f. given by

$$f(x) = \frac{1}{\theta_2} \exp \left[-\frac{(x - \theta_1)}{\theta_2} \right], x > \theta_1, \theta_1 \in R, 0 < \theta_2 < \infty.$$

Find the maximum likelihood estimates of θ_1 and θ_2 . (10%)

參考用

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