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(All the calculations and justification process should be clear; otherwise, no point will be given.)

1. **(20 points)** AngelA brand company plans to promote a new product through two ways: on-line media and newspaper. Suppose the expected sales amount has the following relation with s times appearance in on-line media and n lines in newspaper:

$$f(s,n) = s^3 n$$

Further, the expense for each appearance in on-line media costs \$40,000 and each line spent in newspaper costs \$400. Suppose this company has a total 400,000 budget.

(a) Assist AngelA to allocate spending in these two ways in order to maximize his sales amount. **(10 points)**

(b) Can you mathematically confirm that the answer you get is the local maximum from the second order condition? Justify your answer. **(10 points)**

2. **(10 points)** An electricity company has 1 million continuous income stream with continuous compounding interest rate 5%. How much will it be worth 10 years from now?

3. **(25 points)** IGoStrong is an online drug retailer selling health supplements. Vitamin B is one of its popular products. Demand for such a product is very stable over time. Due to the limited shelf space, this retailer needs to determine an order amount of Vitamin B each time (Several orders can happen in one year) by considering three cost functions. Some notations are firstly defined.

$D =$ annual demand of Vitamin B

$s =$ setup cost each time

$c =$ cost per bottle of Vitamin B

$h =$ holding cost per bottle of Vitamin B kept on the shelf for one year.

$q =$ order amount each time

The first cost function is the annual setup cost $S(q) = \frac{D}{q}s$. The second cost function is the annual variable cost $V = cD$ and the third cost function is the annual holding cost $H(q) = \frac{q}{2}hc$. Now, we acquire more detailed information that the annual demand is

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120,000 bottles per year, the setup cost each time is \$100, and the cost of each bottle is \$3, and the holding cost per bottle kept in the shelf for one year is \$0.6.

(a) Evaluate IGoStrong's optimal order amount each time to minimize the annual total cost. **(10 points)**

(b) Suppose the Vitamin B supplier considers to provide discount to IGoStrong if more order amount each time from IGoStrong appears. For order quantity between 0-5000 units, he will still keep the unit cost to be \$3. If order quantity each time is between 5,000-10,000, the unit cost drops to \$2.96. If the order quantity each time is over 10,000, the unit cost is \$2.92. Determine IGoStrong's optimal order amount each time and calculate the annual total cost. **(15 points)**

4. **(25 points)** Suppose two brand companies competing the sales of a product (company A is the leader and company B is the follower; e.g., decision is made firstly by company A and then by Company B). That is, follower B's order decision is based on leader A's order decision. The sales price is affected depending on the supply level (e.g., supply quantity) from both companies (q_A from Company A and q_b from Company B), and has the price functions $P_A = 6000 - q_A - q_B$ for Company A and $P_B = 6000 - q_A - q_B$ for company B. Standing from the leading company A's point of view, in this case, what is the supply quantity he should provide to maximize his own revenue?

5. **(20 points)** A brand company, AngelC, recently launches a new product. Whether a customer buying this new product depends on if he can accept the retail price. That is, if a potential buyer can accept the retail price up to \$30 (referred to as reservation price) and a brand company eventually determines the retail price to be $p = 25$, then this potential buyer will purchase it. Otherwise when the reservation price for another potential buyer is \$22, then he will not purchase it. Potential buyers' reservation price ξ is continuously distributed in \$20 to \$40 with a probability function $f(\xi) = 0.2 - 0.005\xi$, where $\xi \in (20,40)$. Suppose the market size of potential buyers is 2000 people. That is, for example, the total number of potential buyers whose reservation is allocated in the range between \$28 to \$32 is $2000 \int_{28}^{32} f(\xi)\, d\xi$.

(a) List the revenue if AngelC company tags a retail price $p = 30$. **(10 points)**

(b) What is the optimal retail price which can bring the maximum revenue to this brand company? Why? **(10 points)**

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