考試科目	數理統計學	所別	統計學系 4141	考試時間	2 月 22 日(六) 第三節

1. (15%) Consider an infinite sequence of independent tosses of a coin that comes up heads with probability 0.2.

   (a) (5%) Compute the probability that the 5th head appears on the 10th toss.

   (b) (10%) Let X be such that the first heads appears on the Xth toss. In other words, X is the number of tosses required to obtain a heads. Compute the expectation E[X].

2. (50%) Consider a random sample of size 2, X_1, X_2, from the uniform distribution over the interval $(0, \theta)$ for $\theta > 0$.

   (a) (10%) Find the p.d.f. of the sample range $R = |X_1 - X_2|$.

   (b) (10%) Find an unbiased sufficient estimator of θ, denoted as U, and the p.d.f. of U.

   (c) (10%) Find the maximum likelihood estimator (MLE) of θ, denoted as T.

   (d) (10%) It can be seen in (c) and (d) that the MLE T is a function of the sufficient estimator U. Show the following general result: Let $X_1, \ldots, X_n$ be a random sample from a distribution that has pdf $f(x;\theta), \theta \in \Omega$. If a sufficient statistic U for θ exists and if a MLE T also exists uniquely, then T is a function of U.

   (e) (10%) Which one is a better estimator for θ? R, U or T? Please consider at least two criterion for comparison and explain the results.

3. (10%) Suppose $X \sim Bin(n_X, \pi_X), Y \sim Bin(n_Y, \pi_Y)$ and X, Y are independent. Derive the conditional distribution of X given $X + Y$.

4. (10%) Let $X_1, \ldots, X_n$ be a random sample from a gamma distribution with known parameter α_0 and unknown $\beta > 0$. Construct a confidence interval for β.

5. (15%) If $X_1, \ldots, X_n$ is a random sample from a distribution with the following p.d.f.

$$f(x;\theta) = \begin{cases} \frac{1}{2}\theta^3 x^2 exp\{-\theta x\}, & 0 < x < \infty, \\ 0 & \text{elsewhere} \end{cases}$$

where $0 < \theta < \infty$. Find the unbiased minimum variance estimator of θ.

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