國立政治大學 103 學年度碩士班招生考試試題

考試科目	基礎數學	所別	統計學系 414	考試時間	2月22日 第1節 (六)

(15 pts) Q1. Given that $\lim_{x \to a} x = a$, prove $\lim_{x \to a} x^2 = a^2$, where a is any real number, by using the precise definition($\varepsilon - \delta$ notation) of a limit.

(15 pts) Q2. If *f* is a continuous function on an interval, and if *a* is any number in that interval , then the function defined on the interval as follows is an antiderivative of *f*:

$$F(x) = \int_a^x f(t)dt$$

(10 pts) Q3. Decide whether $\int_1^\infty \frac{\sin x + 3}{\sqrt{x}} dx$ converges or diverges. Show your work.

(10 pts)Q4. Consider the function $f(x, y) = x^{1/3} y^{1/3}$. Show that the partial derivatives $f_x(0,0)$ and $f_y(0,0)$ exist, but that *f* is not differentiable at (0, 0).

(15 pts) Q5. Let **W** be an m-dimensional subspace of $\mathbf{R}^n$ with orthonormal basis {$\mathbf{X}_1$, $\mathbf{X}_2$, ..., $\mathbf{X}_m$}. Then every vector **X** in $\mathbf{R}^n$ can be written as **X** = **Z** + **Y** where **Z** is in **W** and **Y** is orthogonal to every vector in **W**.

(15 pts) Q6. If **L**: V $\rightarrow$ W is a linear transformation, then dim(ker L)+dim(range L)=dim V, assuming that 1 $\leq$ ker L $\leq$ dim V.

Q7. (5 pts) (a) If **A** is a nxn matrix, show that if **X** and **Y** are vectors in $\mathbf{R}^n$, then

$$(\mathbf{AX}) \cdot \mathbf{Y} = \mathbf{X} \cdot (A^T \mathbf{Y})$$

(15 pts) (b) If **A** is a symmetric matrix, then eigenvectors that belong to distinct eigenvalues of **A** are orthogonal.

備註	試題隨卷繳交