國立臺灣師範大學 103 學年度碩士班招生考試試題

科目：**基礎數學** 適用系所：**數學系**

注意：1.本試題共 2 頁，請依序在答案卷上作答，並標明題號，不必抄題。2.答案必須寫在指定作答區內，否則不予計分。

1. (10 points) Find a and b such that the following function f is differentiable everywhere.

$$f(x)=\begin{cases} ax^3, & x\le 3; \\ 2x^2+b, & x>3. \end{cases}$$

2. (10 points) Find the following integrals:

(1) $\displaystyle\int \frac{x-1}{x^3-5x^2+7x-3}\,dx$ (2) $\displaystyle\int_0^2\int_x^2 x^{n-2}\sqrt{1+y^n}\,dydx, n\in\mathbb{N}, n\ge 2.$

3. (10 points) Find the extreme of $f(x,y)=x^2+2y^2-3x+1$ subject to the constraint $x^2+y^2\le 10$. Note: $\sqrt{10}\simeq 3.162$.

4. (10 points) Evaluate the following iterated integral by converting to polar coordinates, where a is positive.

$$\int_0^{2a}\int_0^{\sqrt{2ax-x^2}} xy\,dydx.$$

5. (10 points) Evaluate the following line integral

$$\int_C y^2dx+xydy,$$

where the oriented clockwise curve C is the boundary of the region lying between the graph of $y=0$, $y=\sqrt{x}$ and $x=9$.

6. Let $A=\begin{bmatrix} 1 & 1 & 3 & 0 \\ -1 & 1 & 1 & 2 \\ 0 & 1 & 2 & -1 \\ 2 & -1 & 0 & 6 \end{bmatrix}$ and $\mathbf{b}_1=\begin{bmatrix} 3 \\ -1 \\ 1 \\ 7 \end{bmatrix}$, $\mathbf{b}_2=\begin{bmatrix} -1 \\ 1 \\ 2 \\ 1 \end{bmatrix}$, $\mathbf{b}_3=\begin{bmatrix} 20 \\ -14 \\ 3 \\ 31 \end{bmatrix}$, $\mathbf{b}_4=\begin{bmatrix} 23 \\ 11 \\ 19 \\ 37 \end{bmatrix}$.

(a) (5pts) Compute $\det(A)$.

(b) (8pts) For each $\mathbf{b}_i$, determine whether the system $A\mathbf{x}=\mathbf{b}_i$ is consistent or not.

(c) (7pts) Find an orthonormal basis for the column space and null space of A, respectively.

（背面尚有試題）

7. Let $\mathcal{P}_2(\mathbb{R})$ denote the vector space consist of all polynomials with coefficients from $\mathbb{R}$ having degree less or equal to 2. Suppose that T is a linear operator on $\mathcal{P}_2(\mathbb{R})$ defined by

$$T(f(x)) = f(1) + f'(0)x + (f'(0) + f''(0))x^2.$$

(a) (5pts) Let A be the matrix representation of T in the ordered basis $\{x^2, x, 1\}$. Compute A.

(b) (10pts) Show that A is diagonalizable.

(c) (5pts)Find an orthogonal matrix Q such that $Q^{-1}AQ$ is a diagonal matrix.

8. (10pts) Suppose A is an $m \times n$ matrix with rank m and $\mathbf{v}_1, \mathbf{v}_2, \cdots, \mathbf{v}_k \in \mathbb{R}^n$ are vectors with $\mathrm{Span}(\mathbf{v}_1, \mathbf{v}_2, \cdots, \mathbf{v}_k) = \mathbb{R}^n$. Prove that $\mathrm{Span}(A\mathbf{v}_1, A\mathbf{v}_2, \cdots, A\mathbf{v}_k) = \mathbb{R}^m$.

(試題結束)