注意事項：本試題共有 20 題複選題，每題正確答案至少一個(一個或超過一個)，須全部答對才能取得該題分數，未完全答對不給分，答錯不倒扣，每題配分 5 分，合計 100 分。

請使用答案卡作答

1. Given the following probability distribution:

	Y		
X	0	1	2
0	0.1	0	0.2
1	0.1	0.1	0
2	0	0.3	0.2

Then (A) $\Pr(Y \leq 1) = 0.2$　(B) $\Pr(X < 1 \mid Y < 1) = 0.5$　(C) $\Pr(X = Y \mid Y \geq 1) = 0.5$

(D) $\Pr(X > 1) = 0.3$　(E) $\Pr(Y > 1 \mid X > 1) = 0.4$

2. Let X have a Poisson distribution with $\lambda = 2$. Then

(A) $E(X) = 2$　(B) $\text{Var}(X) = 4$　(C) $\Pr(X \geq 1) = 1 - e^{-2}$

(D) $\Pr(X \leq 1) = e^{-2}$　(E) $\Pr(X > 10) = 0$

3. Let $X_1, X_2, \ldots, X_{100}$ constitute a random sample of size 100 from Bernoulli(p). Then

(A) $E(X_2) = 100p$　(B) $E(X_1) = E(X_{100})$　(C) $\text{Var}(X_3) = p(1-p)$

(D) $\text{Var}(X_4) = 100p(1-p)$　(E) $E(X_{10} X_{20} X_{30}) = E(X_{10})\, E(X_{20})\, E(X_{30})$

4. Let $X_1, X_2, \ldots, X_{100}$ constitute a random sample of size 100 from Bernoulli(p). Which of the following functions are statistics?

(A) $\sum_{i=1}^{100} X_i$　(B) $\sum_{i=1}^{100} X_i / 100$　(C) $(X_{50} - \sum_{i=1}^{100} X_i / 100)^2$

(D) p　(E) $p(1-p)$

5. Let $X_1, X_2, \ldots, X_{100}$ constitute a random sample of size 100 from Bernoulli(p) and $Y = \sum_{i=1}^{100} X_i$. Then

(A) Y is binomial $(100, p)$　(B) $\Pr(Y = 100) = (1-p)^{100}$　(C) $E(Y) = p$

(D) $\text{Var}(Y) = 100\, p(1-p)$　(E) $\overline{X} = Y / 100$ is approximately $N(p, p(1-p))$.

6. Let W have a gamma (α, β) distribution. Then

(A) $E(W) = \alpha$　(B) $E(W) = \beta$　(C) $Var(W) = \alpha\beta$　(D) $Var(W) = \alpha\beta^2$　(E) $Var(W) = \alpha^2\beta$

7. Let X and Y have a trinomial distribution (n, p_1, p_2). Then

(A) X is $b(n, p_1)$

(B) Y is $b(n, p_2)$

(C) the correlation coefficient of X and $Y = p_1p_2/[(1-p_1)(1-p_2)]$

(D) the correlation coefficient of X and $Y = \sqrt{p_1p_2/[(1-p_1)(1-p_2)]}$

(E) the correlation coefficient of X and $Y = -\sqrt{p_1p_2/[(1-p_1)(1-p_2)]}$

8. Let the joint distribution of X and Y be $N(\mu_X, \mu_Y, \sigma_X^2, \sigma_Y^2, \rho)$. Then

(A) X is $N(\mu_X, \sigma_X^2)$.

(B) $E(Y|x) = \mu_Y + \rho\dfrac{\sigma_X}{\sigma_Y}(x - \mu_X)$.

(C) $E(Y|x) = \mu_Y + \rho\dfrac{\sigma_Y}{\sigma_X}(x - \mu_X)$.

(D) $Var(Y|x) = \sigma_Y^2(1-\rho^2)$.

(E) $Var(Y|x) = \sigma_X^2(1-\rho^2)$.

9. Let the variance-covariance matrix of X_1, X_2, X_3, X_4 be given by

$$\begin{bmatrix} \sigma_1^2 & \sigma_{12} & 0 & 0 \\ \sigma_{12} & \sigma_2^2 & 0 & 0 \\ 0 & 0 & \sigma_3^2 & \sigma_{34} \\ 0 & 0 & \sigma_{34} & \sigma_4^2 \end{bmatrix}.$$

Then

(A) $Var(X_1 + 2) = \sigma_1^2 + 2$

(B) $Var(X_1 - X_3) = \sigma_1^2 - \sigma_3^2$

(C) $Var(2X_1 - X_2) = 4\sigma_1^2 + \sigma_2^2 - 4\sigma_{12}$

(D) $Cov(2X_1 + 2X_2, X_3 + X_4) = 0$

(E) $Cov(X_1 + X_3, 2X_2 - X_4) = 2\sigma_{12} - \sigma_{34}$

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【可使用計算機】＊作答前請先核對試題、答案卷(試卷)與准考證之所組別與考科是否相符！！

10. Let $X_1, X_2, \ldots, X_n$ constitute a random sample of size n from $N(\mu, \sigma^2)$. Let $\overline{X}$ and $S^2 = \dfrac{\sum_{i=1}^{n}(X_i - \overline{X})^2}{n-1}$ denote, respectively, the sample mean and sample variance. Then

(A) $\overline{X}$ is $N(\mu, \sigma^2/n)$.

(B) $\sum_{i=1}^{n}(X_i - \mu)^2 / \sigma^2$ is $\chi^2(n-1)$.

(C) $\sum_{i=1}^{n}(X_i - \overline{X})^2 / \sigma^2$ is $\chi^2(n)$.

(D) $\sqrt{n}\,(\overline{X} - \mu) / S$ has a t distribution with $(n-1)$ degrees of freedom.

(E) $n(\overline{X} - \mu)^2 / S^2$ has an F distribution with 1 and n degrees of freedom.

11. To construct a 95% confidence interval for the normal population mean μ, based on a sample of size 80, when σ^2 is unknown, we need the information of

(A) the sample mean

(B) the sample standard deviation

(C) the 95th percentile of the t distribution with 80 degrees of freedom

(D) the 97.5th percentile of the t distribution with 79 degrees of freedom

(E) the 97.5th percentile of the t distribution with 80 degrees of freedom

12. Select correct items about testing H_0: $\mu = \mu_0$ against H_1: $\mu \neq \mu_0$ for the normal population mean μ at the α level of significance:

(A) Pr (critical region | H_0) = α.

(B) Pr (critical region | H_1) = α

(C) Pr (reject H_0 | H_0) = power

(D) Pr (accept H_0 | H_1) = power

(E) Pr (reject H_0 | H_1) = power

13. Let X be a continuous random variable and $F(x)$ its distribution function. Let $Z = F(X)$. Then the distribution of Z is

(A) N(0,1)　(B) exponential(1)　(C) uniform (0,1)　(D) Bernoulli(0.5)　(E) χ^2 (1)

14. If Z_1 and Z_2 are independent and identically distributed as N(0,1). Then

(A) $Z_1 - Z_2$ is N(0,2).

(B) $Z_1^2 + Z_2^2$ is $\chi^2(2)$.

(C) $Z_1 / \sqrt{Z_2^2}$ is $t(2)$.

(D) $Z_2 / \sqrt{Z_1^2}$ is $t(1)$.

(E) Z_1^2 / Z_2^2 is $F(1,1)$.

15. To show if a statistic is a sufficient statistic for a parameter, we can use

(A) central limit theorem (B) Rao-Blackwell theorem (C) factorization theorem

(D) Neyman-Pearson theorem (E) Cochran's theorem

16. To find a best critical region for testing a simple hypothesis against an alternative simple hypothesis, we can use

(A) central limit theorem (B) Rao-Blackwell theorem (C) factorization theorem

(D) Neyman-Pearson theorem (E) Cochran's theorem

17. Let $X_1, X_2, \ldots, X_{n_1}$ denote a random sample of size n_1 from $N(\mu_1, \sigma_1^2)$ and $Y_1, Y_2, \ldots, Y_{n_2}$ denote another random sample of size n_2 from $N(\mu_2, \sigma_2^2)$. Let $\bar{X}$ and S_X^2 be the sample mean and the sample variance for the former and $\bar{Y}$ and S_Y^2 be those for the latter.

(A) To test $H_0\colon \sigma_1^2 = \sigma_2^2$ against $H_1\colon \sigma_1^2 \neq \sigma_2^2$, use S_X^2 / S_Y^2, which is distributed as $F(n_1, n_2)$.

(B) Given that $\sigma_1^2 = \sigma_2^2 = \sigma^2$, we use $t = \dfrac{(\bar{X} - Y)}{S_p\sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}}$, where $S_p^2 = \dfrac{\sum_{i=1}^{n_1}(X_i - \bar{X})^2 + \sum_{i=1}^{n_1}(Y_i - \bar{Y})^2}{n_1 + n_2 - 2}$, for testing H_0: $\mu_1 = \mu_2$ against H_1: $\mu_1 \neq \mu_2$.

(C) Given that $\sigma_1^2 = \sigma_2^2 = \sigma^2$, we can show that $(n_1 + n_2 - 2) S_p^2 / \sigma^2$ is distributed as $\chi^2(n_1 + n_2 - 2)$.

(D) Under H_0: $\mu_1 = \mu_2$, the test statistic presented in (B) is distributed as $t(n_1 + n_2 - 2)$.

(E) The test statistic presented in (B) results from the likelihood ratio test.

18. To test if the observed data come from a hypothesized distribution, use

(A) the binomial test (B) the normal test (C) the t test

(D) the χ^2 test (E) the F test

19. Select correct items about test of homogeneity and test of independence in the two-way contingency table analysis:

(A) Both tests have the same null hypothesis.

(B) Both tests have the same test statistic.

(C) Both tests have the same sampling distribution.

(D) Both tests have the same sampling procedure.

(E) Both tests apply for large samples.

20. Given the following output for the regression model

$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \beta_3 X_{3i} + \varepsilon_i,\ \ i = 1, 2, \ldots, 40,$

Predictor	Parameter Estimate	Standard Error	t	p
Intercept	249.32	1.455	171.35	< 0.0001
X_1	1.46	2.058	0.71	0.4832
X_2	13.94	2.446	5.70	< 0.0001
X_3	19.26	1.922	10.02	< 0.0001

ANOVA Table

Source	df	SS	MS	F	p
Regression	3	2794.39	931.46	43.98	< 0.0001
Error	36	762.30	21.18		
Total	39	3556.69			

(A) $F = 43.98$ can be used to test H$_0$: $\beta_1 = \beta_2 = \beta_2$.

(B) we reject H$_0$: $\beta_1 = 0$, holding X_2 and X_3 constant.

(C) we reject H$_0$: $\beta_2 = 0$, holding X_1 and X_3 constant.

(D) the p value of 0.4832 is based on the t distribution with 39 degrees of freedom.

(E) $R^2 = 2794.39 / 3556.69$.