國立臺北大學 103 學年度碩士班一般入學考試試題

系（所）組別：統計學系

科　　目：數理統計

□可 ☑不可使用計算機

1. (25%) Let $X_1, X_2, \ldots, X_n$ be iid from a $N(0,\theta)$ distribution. We want to estimate the standard deviation $\sqrt{\theta}$.

(a) Find the density function of $|X_i|$.

(b) Find the constant c so that $\overline{Y_n} = c\Sigma|X_i|$ is unbiased. Is $\overline{Y_n}$ an UMVUE?

(c) Find the values of a and b of the following

$$\sqrt{n}\left(\overline{Y_n} - a\right) \to N(0,b).$$

(d) Considering the following hypothesis testing

$$H_0 : \theta = 1 \quad \text{v.s.} \quad H_1 : \theta \neq 1.$$

Use the likelihood ratio test to find the test statistic.

(e) Determine the rejection area of the test statistic in (d) with a level α.

2. (25%) Let a random sample of size n be taken from a distribution of the discrete type with probability mass function $f(x;\theta) = \frac{1}{\theta}, x = 1,2,\ldots,\theta$, zero elsewhere, where θ is an unknown positive integer.

(a) Let $Y_n = \max\{X_1, X_2, \ldots, X_n\}$. Find the probability mass function of Y_n.

(b) Find an unbiased estimate for θ.

(c) Find the conditional distribution of $Y_1 = \min\{X_1, X_2, \ldots, X_n\}$ given Y_n.

3. (25%) Let X and Y are two random variables with CDF $F_X(t)$, $F_Y(t)$, respectively.

(a) Define the term "X is stochastically larger than Y" through CDF.

(b) Let X_λ~Poisson(λ). Is there any stochastical order between X_a and X_b, if $a > b$?

(c) Let X_β~Gamma(3, β). Is there any stochastical order between X_a and X_b, if $a > b$?

4.(25%) Let $X_1, X_2, \ldots, X_n$ be a random sample from a population with pdf or pmf $f(x|\theta)$. Given two distinct constants, θ_0, θ_1, we would like to test H_0: $\theta = \theta_0$ vs. H_1: $\theta = \theta_1$.

(a) Describe the Neyman-Pearson lemma.

(b) Let $f(x|\theta) \sim$ binomial(4, θ). Use the random sample X_1, X_2 to find a most powerful test for H_0: $\theta = 1/2$ vs. H_1: $\theta = 1/4$ at significant level 11%.

(c) Let $f(x|\theta) \sim$ Gamma(2, θ). Use the random sample of size n to find a most powerful test for H_0: $\theta = 1$ vs. H_1: $\theta = 2$ at significant level 5%.

試題隨卷繳交