國立臺北大學 103 學年度碩士班一般入學考試試題

系（所）組別： 金融與合作經營學系

科　　　目： 統計學

第 1 頁 共 1 頁

☑可 ☐不可使用計算機

一、(15%)

Assume that X_1, X_2, …, and X_n are independent random variables and that $\mathrm{E}(X_i) = \mu$ and $\mathrm{Var}(X_i) = \sigma_i^2$ with $i = 1, 2, \ldots, n$. Let $\bar{X} = \sum_{i=1}^{n} X_i/n$.

(1) What are $\mathrm{E}(\bar{X})$ and $\mathrm{Var}(\bar{X})$? (5%)

(2) Find an unbiased estimator of $\mathrm{Var}(\bar{X})$. And show that. (5%)

(3) Let $\lim_{n\to\infty} \frac{\sum_{i=1}^{n}\sigma_i^2}{n} = \bar{\sigma}^2 < \infty$. Show that $\bar{X}$ is a consistent estimator of μ. (5%)

二、(15%)

Assume that $y_i = \beta x_i + \varepsilon_i$ and $\varepsilon_i \sim N(0, \sigma^2)$ with $i = 1, 2, \ldots, n$ are independent random variables.

(1) What is the OLS (*Ordinary Least Squares*) estimator of β? (5%)

(2) Prove that OLS estimator of β is unbiased. (5%)

(3) Find an unbiased estimator of σ^2. And show that. (5%)

三、(20%)

There are three stocks 1, 2 as well as 3; and their vector of return rates is

$\mathbf{R}=\begin{bmatrix} R_1 \\ R_2 \\ R_3 \end{bmatrix} \sim N(\boldsymbol{\mu} = \begin{bmatrix} 0.10 \\ 0.10 \\ 0.08 \end{bmatrix}, \boldsymbol{\Sigma} = \begin{bmatrix} 0.09 & -0.09 & 0 \\ -0.09 & 0.09 & 0 \\ 0 & 0 & 0.07 \end{bmatrix})$. Let $\mathbf{w} = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix}$, where $w_3 = 1 - w_1 - w_2$, is the weight vector on the three stocks for a portfolio **P**. Namely, the return rate of **P** is $R_P = \mathbf{w}^{\mathrm{T}}\mathbf{R}$, its expected return rate is $\mathrm{E}(R_P) = \mathbf{w}^{\mathrm{T}}\boldsymbol{\mu}$ and the variance of its return rate is $\sigma_P^2 = \mathbf{w}^{\mathrm{T}}\boldsymbol{\Sigma}\mathbf{w}$, where the superscript T denotes the matrix transpose operation

(1) Find a weight vector $\mathbf{w}$ such that R_P has the minimum variance. (10%)

(2) Let $\mathbf{w} = [0, \ 0.5, \ 0.5]^{\mathrm{T}}$ and the observed return rate $R_P = 0.18$. Is R_P significantly greater than the expected return rate $\mathrm{E}(R_P)$? And explain why or why not ? (10%)

四、(10%)

A cell phone company surveys the way of its customers buying products. There are 30% buying from the website, 60% buying from the shops, and 10% buying from other ways. 50% customers of the group buying from the website purchase high-level phones; 60% customers of the group buying from shops purchase high-level phones; besides, 40% customers of the group buying from other ways purchase high-level phones. What is the probability when a customer purchasing high-level phones buying from the website?

五、(14%)

A student is assigned to generate a set of random variables in an exercise. He has heard that the process of generating random variables can be implemented by inverse transformation from uniform distribution R$\sim$U(0,1). Please transform R=0.4 and R=0.8 to the mapping value of following probability density functions: (You have to write the transforming process as clear as possible)

$$f(x) = \begin{cases} x & 0 \le x \le 1 \\ 2 - x & 1 < x \le 2 \\ 0 & \text{otherwise} \end{cases}$$

六、(13%)

Independent sample X_1, X_2, X_3 and $Y_1, Y_2, \ldots, Y_5$ are selected from $N(\mu, \sigma^2)$. $V = 3X_1 - 4X_2 + 2X_3$, $W = \sum_{i=1}^{5}(Y_i - \bar{Y})^2$,

$\bar{Y} = \frac{1}{5}\sum_{i=1}^{5} Y_i$ Please explain how to use V and W to build a t distribution. (You have to indicate the degree of freedom, and write the process as clear as possible).

七、(13%)

Throw a fair coin 10000 times. What is the approximate probability that the tail appears over 5100 times?

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