國立高雄大學 102 學年度研究所碩士班招生考試試題

科目：機率論
考試時間：100 分鐘

系所：
統計學研究所(統計組)
本科原始成績：100 分

是否使用計算機：否

1. (10%) Let X and Y be independent uniform$(0,1)$ random variables. Please compute the following probabilities: (a) $P(|X-Y| \leq 0.5)$; (b) $P(Y \geq X \mid Y \geq 0.5)$.

2. (10%) Let X be a Poisson(λ) random variable. Use Chebyshev's inequality to derive the following inequalities: (a) $P(X \leq \lambda/2) \leq 4/\lambda$; (b) $P(X \geq 2\lambda) \leq 1/\lambda$.

3. (10%) Let X be uniform$(0,1)$ and Y be exponential(λ) random variables. If X and Y are independent, then what is the density function of $Z = X + Y$?

4. (15%) Let X and Y be independent gamma(α_1, λ) and gamma(α_2, λ) random variables, respectively. Is Y/X independent of $X+Y$? Please verify your answer.

5. (15%) Let X and Y be independent exponential(λ) random variables. Let $Z = \max(X, Y)$. Please compute $E(Z)$ and $Var(Z)$.

6. (10%) Let $M_X(t) = p^n(1 - e^t(1-p))^{-n}$ be the moment generating function of a random variable X, where $0 < p < 1$ and $t < -\ln(1-p)$. Please compute $E(X)$ and $Var(X)$ by using $M_X(t)$.

7. (15%) Let U and V be independent normal$(0,1)$ random variables. Let $Z = \rho U + \sqrt{1-\rho^2}\,V$, where $|\rho| < 1$. Please compute (a) the joint density of $X = \mu_1 + \sigma_1 U$ and $Y = \mu_2 + \sigma_2 Z$, where σ_1 and $\sigma_2 > 0$, and (b) the conditional density of $Y|X = x$.

8. (15%) Let X_n be a gamma(n, λ) distribution with mean n/λ, where n is an integer and $\lambda > 0$. Please use central limit theorem to derive the limiting distribution of $(\lambda X_n - n)/\sqrt{n}$ as $n \to \infty$.