

國立中山大學 102 學年度碩士暨碩士專班招生考試試題

科目名稱：統計學【財管系碩士班】

題號：443003

※本科目依簡章規定「不可以」使用計算機

共 4 頁第 1 頁

說明：

統計學[財管系碩士班]試題共有兩部份，第一部份為填充題 50 分，第二部份為問答题 50 分，請依序作答，總計共 100 分。

填充題答案全對才給分。你只需要填入最後的完整答案，”請勿”寫出計算過程。

例如：你的第一個空格答案若為“ $a+b$ ”，答案請填寫 (1) $a+b$ 。

填充題答案卷請製作如下：

(1)	(2)	(3)
(4)	(5)	(6)
(7)	(8)	(9)
(10)		

第一部份填充題：

共 10 格，每一個空格 5 分，共計 50 分。(請務必依據答案欄要求的格式，以及要求作答的方式)

Question 1.

We've collected data on the percentage of the population of each county in Taiwan that is considered obese, using clinical measures, and that has been diagnosed with diabetes. We have constructed two random variables, *Obesity* and *Diabetes*, of the following forms (note: how the categories are constructed is irrelevant),

Obesity = 1 if percent obese is <18.6

2 if percent obese is $18.6 - 23.5$

3 if percent obese is >23.5

Diabetes = 1 if percent w/ diabetes is <6.4

2 if percent w/ diabetes is $6.4 - 7.6$

3 if percent w/ diabetes is >7.6

The following table reflects the joint probability function of our two variables:

	<i>Obesity</i> = 1	<i>Obesity</i> = 2	<i>Obesity</i> = 3
<i>Diabetes</i> = 1	0.017	0.000	0.034
<i>Diabetes</i> = 2	0.0165	0.278	0.189
<i>Diabetes</i> = 3	0.0165	0.122	0.327

What is the probability that *Obesity* = 2, say $\Pr(\text{Obesity}=2) = \underline{(1)}$, what is the conditional probability of *Diabetes*=1 given that *Obesity* = 1, say $\Pr(\text{Diabetes}=1 / \text{Obesity}=1) = \underline{(2)}$. Calculate conditional expectation of *Diabetes* given that *Obesity* = 1, say $E(\text{Diabetes} / \text{Obesity}=1) = \underline{(3)}$, and covariance of *Diabetes* and *Obesity*, say $\text{Cov}(\text{Obesity}, \text{Diabetes}) = \underline{(4)}$.

Question 2.

A manufacturer general manager claims that his brand of DVD player has an average life expectancy of 6 years and 6 months with a standard deviation of 1 year and 3 months. Assume that the life expectancy is normally distributed. Randomly selecting one DVD player from this brand, find the probability of its life expectancy (years): $\Pr(6 < Y < 8) = \underline{(5)}$. Assume you decide to test 100 DVDs of this brand, the average life in these samples is 5 years and the sample standard deviation is 2 years. Calculate a 95% level of confidence interval for the average life (in years) = (6) (四捨五入至小數點以下第 3 位).

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共 4 頁 第 2 頁

Question 3.

You are estimating the linear regression model by Ordinary Least Square (OLS) method, setting the basic model $Y_i = b_0 + b_1 X_i + \varepsilon_i$ (ε_i is error term), and considering the following data,

Y	8	3	4	2	-2
X	5	3	-1	2	0

Find the estimator of $b_1 = \underline{(7)}$ (四捨五入至小數點以下第 3 位). Then assume given sum of squared residuals is 30.772, the variance of the regression = (8)。(四捨五入至小數點以下第 3 位)

Question 4.

Suppose you have estimated a simple regression model that explains household food expenditures (in dollars) as a function of household income using 1000 observations:

	Coefficient	Std. error	T-stat.	P-value
<i>Intercept</i>	83.41	(甲)	1.92	0.062
<i>INCOME</i>	10.20	2.09	(乙)	0.000

Find the value of (甲) = (9) (四捨五入至小數點以下第 3 位). Construct a 95% interval estimate for coefficient of *INCOME* = (10)。(四捨五入至小數點以下第 3 位)

[最後有兩張可查詢的表]

第二部分—問答題，共 4 個小題，每小題的配分在題號前，共計 50 分。

- (5 分) Suppose that X and Y are random variables. Please derive $Cov(X, Y)^2 \leq Var(X)Var(Y)$.
- (10 分) A random sample $X_1, X_2, \dots, X_n$ of size n is taken from $N(\mu, \sigma^2)$ where μ is a real number and σ^2 is finite. Please find the Cramer-Rao lower bound (C.R.L.B.).
- (15 分) Let future stock price S_T as a lognormal random variable, i.e.,
 $\ln S_T \sim N(\ln S_0 + (r - \frac{1}{2}\sigma^2)T, \sigma^2 T)$, where S_0, K, r, σ, T are all constant numbers. Please prove
 that $e^{-rT} E[Max(S_T - K, 0)]$ is equal to $S_0 N(d_1) - Ke^{-rT} N(d_2)$, where $d_1 = \frac{\ln(S_0/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}$
 and $d_2 = d_1 - \sigma\sqrt{T}$.
- (20 分) The discrete uniform distribution is known as the "equally likely outcomes" distribution. Suppose X is a discrete uniform random variable. If we let the values of the random variable be $1, 2, \dots, N$, then the probability mass function (pmf) $f(x) = 1/N$, where $x = 1, 2, \dots, N$. Please prove the following identities.

(a) $E(X) = \frac{N+1}{2}$ (b) $Var(X) = \frac{N^2-1}{12}$ (c) $M_X(t) = \frac{e^t(1-e^{Nt})}{N(1-e^t)}$ (d) $E(X^n) = \sum_{x=1}^N \frac{x^n}{N}$

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共 4 頁第 3 頁

A.5 F分配之臨界值 (續)

$f_{0.05}(v_1, v_2)$

v_2	v_1									
	10	12	15	20	24	30	40	60	120	∞
1	241.9	243.9	245.9	248.0	249.1	250.1	251.1	252.2	253.3	254.3
2	19.40	19.41	19.43	19.45	19.45	19.46	19.47	19.48	19.49	19.50
3	8.79	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53
4	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63
5	4.74	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.36
6	4.06	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67
7	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23
8	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.10	2.97	2.93
9	3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71
10	2.98	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54
11	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.40
12	2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30
13	2.67	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21
14	2.60	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13
15	2.54	2.48	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07
16	2.49	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01
17	2.45	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.01	1.96
18	2.41	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92
19	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88
20	2.35	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84
21	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81
22	2.30	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78
23	2.27	2.20	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1.76
24	2.25	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73
25	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71
26	2.22	2.15	2.07	1.99	1.95	1.90	1.85	1.80	1.75	1.69
27	2.20	2.13	2.06	1.97	1.93	1.88	1.84	1.79	1.73	1.67
28	2.19	2.12	2.04	1.96	1.91	1.87	1.82	1.77	1.71	1.65
29	2.18	2.10	2.03	1.94	1.90	1.85	1.81	1.75	1.70	1.64
30	2.16	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62
40	2.08	2.00	1.92	1.84	1.79	1.74	1.96	1.64	1.58	1.51
60	1.99	1.92	1.84	1.75	1.70	1.65	1.59	1.53	1.47	1.39
120	1.91	1.83	1.75	1.66	1.61	1.55	1.50	1.43	1.35	1.25
∞	1.83	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1.00

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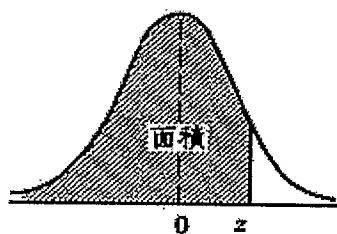
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共 4 頁第 4 頁

A.2 常態曲線下之面積



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0017	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0197	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0352	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0722	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3717	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
-0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641