# 國立中山大學102學年度碩士暨碩士專班招生考試試題

科目名稱：離散數學【資工系碩士班甲組】
※本科目依簡章規定「不可以」使用計算機

題號：434004


*There are 9 problems in this test. Note that you should write down detailed steps for the solution to each problem; otherwise, no credits for that problem will be given.*

1. [9%] Ben works as a computer operator at National Sun Yat-sen University. One evening he finds that fifteen computer programs have been submitted earlier that day for batch processing. In how many ways can Ben order the processing of these programs if (a) [3%] there are no restrictions? (b) [3%] he considers five of the programs higher in priority than the others and wants to process those five first? (c) [3%] he first separates the programs into four of top priority, eight of lesser priority, and three of least priority, and he wishes to process the fifteen programs in such a way that the top-priority programs are processed first and the three programs of least priority are processed last?

2. [12%] A computer science professor has thirteen different programming books on a bookshelf. Six of the books deal with C++, the others with Java. In how many ways can the professor arrange these books on the shelf (a) [3%] if there are no restrictions? (b) [3%] if the languages should alternate? (c) [3%] if all the Java books must be next to each other? (d) [3%] if all the C++ books must be next to each other and all the Java books must be next to each other?

3. [12%] Four hundred coins numbered 1 to 400 are put in a row across the top of a cafeteria table. Four hundred students are assigned numbers (from 1 to 400) and are asked to turn over certain coins. The student assigned number 1 is supposed to turn over all the coins. The student assigned number 2 is supposed to turn over every other coin, starting with the second coin. In general, the student assigned the number $n$, for each $1 \leq n \leq 400$, is supposed to turn over every $n$th coin, starting with the $n$th coin. (a) [4%] How many times will the 400th coin be turned over? (b) [4%] Which coin(s) will be turned over as many times as the 400th coin? (c) [4%] Please show two coins that will be turned over more than 18 times?

4. [15%] Consider parking motorcycles and compact cars in a row of spaces where each cycle requires one space and each compact needs two. Find and solve a recurrence relation for the number of ways to park motorcycles and compact cars in a row of $n$ spaces (a) [5%] if all cycles are identical in appearance, as are the cars, and we want to use up all the $n$ spaces. (b) [5%] if the motorcycles come in two distinct models and the compact cars come in three different colors, and we want to use up all the $n$ spaces. (c) [5%] if all cycles are identical in appearance and the compact cars come in three different colors, and empty spaces are allowed.

5. [12%] Consider the equation $c_1 + c_2 + c_3 + c_4 = 23$ where $-3 \leq c_1$, $-2 \leq c_2$, $-2 \leq c_3 \leq 0$, and $2 \leq c_4$. (a) [4%] Find the generating function for the number of integer solutions to the equation. (b) [3%] The number of integer solutions to the above equation equals to the coefficient of $x^n$ of the above generating function. What is $n$? (c) [5%] Please find the coefficient of $x^{100}$ in the above generating function.

6. [9%] Determine the sequence generated by each of the following exponential generating functions. (a) [3%] $f(x) = 2e^{2x}$ (b) [3%] $f(x) = 5e^{3x} - 3e^{2x}$ (c) [3%] $f(x) = 1/(1-x) - x^2$.

7. [10%] For $n \geq 1$, let $X_n = \{1, 2, 3, ..., n\}$. $P(X_n)$ denotes the power set of $X_n$. (a) [8%] Let $e_n$ be the number of edges in the Hasse diagram for the partial order $(P(X_n), \subseteq)$. Please find a recurrence relation for $e_n$ and then solve the recurrence relation to get $e_n$. (b) [2%] Please find $v_n$, the number of vertices of the diagram in (a).

8. [11%] Please answer the following questions. (a) [4%] Find all generators of the cyclic group $(\mathbf{Z}_8, +)$. (b) [4%] Find all generators of the cyclic group $(\mathbf{Z}_5-\{0\}, *)$. (c) [3%] If $G$ is a cyclic group of order $n$, how many distinct generators does it have?

9. [10%] Please find an encryption function $E$: $\{1, 2, 3, ..., 26\}$ -> $\mathbf{Z}$ such that $E(m_1) + E(m_2) = E(m_1+m_2)$ for every $m_1$, $m_2$ in $\{1, 2, 3, ..., 26\}$ and please also find the decryption function corresponding to $E$.