國立中山大學102學年度碩士暨碩士專班招生考試試題

科目名稱：工程數學【資工系碩士班乙組】　　題號：434002

※本科目依簡章規定「不可以」使用計算機　　

可能會用到電子計算機之計算式所得之數值會加註在題目後；如有需電子計算機計算之數值而題目後未列入，則請詳記列出計算公式與過程，此時之計算結果以計算式為答案，不因電子計算機之所計算數值而加以扣分。例如，題目提示 ln0.8 = -0. 224, 但您的答案為 3ln0.2，或 3(ln0.1+ln2)，均為正確答案。

1. (15%) Solve the Differential Equation:
   $(2x+ye^{xy})dx + (xe^{xy})dy = 0, y(1)=0$
   (Hint: Is this an exact differential equation?)

2. (20%) Laplace Transform
   (a) (12%) Solve the following differential equation by Laplace Transform and Inverse Laplace Transform
   $$y''+2y'+10y=e^{-t}\sin t;\ y(0)=0,\ y'(0)=1$$
   (b) (8%) Let $f(t)=t$ and $g(t)=\cos t$. Find the convolution of f and g (i.e., $f*g$)

3. (25%) Matrices and Linear Systems
   (a) (5%) Given $\mathbf{A}^{-1}=\begin{bmatrix}1 & 2 & -1\\ 3 & 4 & 2\\ 0 & 1 & -2\end{bmatrix}$, $\mathbf{B}^{-1}=\begin{bmatrix}0 & 1 & 1\\ 1 & 0 & 1\\ -2 & 3 & 2\end{bmatrix}$, find $(\mathbf{AB})^{-1}$
   (b) (10%) Solve the following linear system by using Gauss-Jordon elimination:
   $$\begin{cases} x_1 - x_2 + x_3 = 4 \\ 3x_1 + 2x_2 + x_3 = 2 \\ 4x_1 + 2x_2 + 2x_3 = 8 \end{cases}$$
   Please show step-by-step operations; otherwise, no points will be given.
   (c) (5%) Find Rank(A), $\mathbf{A}=\begin{bmatrix}3 & 1 & 4 & 0\\ 1 & 0 & 1 & -2\\ 2 & 1 & 3 & 2\end{bmatrix}$
   (d) (5%) Find matrix **A** representing the linear transformation that maps (x_1, x_2) onto $(2x_1-5x_2, 3x_1+4x_2)$

4. (25%) Eigen value and matrix diagonization
   A finite state machine (FSM) has two states: state S1 and state S2. Let Prob(S1→S2) denote the probability that the machine moves from state S1 to state S2 in a cycle. Assume that the state transition probabilities are fixed in every cycle: Prob(S1→S1)=0.3, Prob(S1→S2)=0.7, Prob(S2→S1)=0.6, Prob(S2→S2)=0.4. Initially, the FSM is in state S1 at cycle 0.

   (a) (5%) Give a matrix **A** representing the state transition probabilities.
   (b) (5%) At the end of the second cycle, what is the probability that the FSM is in state S1? What is the probability that the FSM is in state S2?
   (c) (15%) At the end of the 1000th cycle, what is the probability that the FSM is in state S1? What is the probability that the FSM is in state S2?
   (Hint: You need to convert **A** to a diagonal matrix)

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5. (15%) Fourier Series

Find the Fourier series of the function $f(x)=\begin{cases} -k, -2 < x < 0 \\ k, 0 < x < 2 \end{cases}$, and $f(x+4)=f(x)$.