國立中山大學102學年度碩士暨碩士專班招生考試試題

科目名稱：工程數學【機電系碩士班甲組、乙組、丙組】　　題號：438002

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）　　

I. (35%)

1. (10%) Find all solutions or indicate that no solution exists.

$$\begin{aligned} 4y+z&=0\\ 12x-5y-3z&=34\\ -6x+4z&=8 \end{aligned}$$

2. (15%) Find the eigenvalues and the corresponding eigenvectors. Use the given λ.

$$\begin{bmatrix} 4 & 2 & -2\\ 2 & 5 & 0\\ -2 & 0 & 3 \end{bmatrix}, \lambda=4$$

3. (10%) Experiments show that in a temperature field, heat flows in the direction of maximum decrease of temperature T. Find this direction in general and at the given point P.

$T=\dfrac{z}{x^2+y^2}$, P: (0, 1, 2)

II. (35%)

1. Solve the following ordinary differential equations (ODEs):

(a) (10%) $(1-2x-x^2)y''+2(1+x)y'-2y=0$

(b) (13%) $2xy''+(1+x)y'+y=0$

(c) (12%) $y''+3y'+2y=\begin{cases}0, & \text{if } t<1\\ 1, & \text{if } 1<t<2\\ 0, & \text{if } t>2\end{cases}$ with I.C.: $\begin{cases}y(1)=1\\ y'(1)=-1\end{cases}$

III. (30%)

1. Please answer the following questions:

(a) (5%) What is "Fourier series" used for?

(b) (5%) What is "orthogonality" regarding the Fourier series?

(c) (10%) Please find the Fourier series according to the following periodic rectangular wave:

$$\mathrm{F(x)}=\begin{cases}0 & if\ -2<x<-1\\ k & if\ -1<x<1\\ 0 & if\ 1<x<2\end{cases}, k\ is\ a\ constant$$

2. (10%) What is "partial differential equation (P.D.E)"? Please give an example. (You don't have to solve the P.D.E in your example.)