PROBLEM 1 (40%)

Given a system described by the following differential equation

$$\frac{d^2y}{dt^2} - 5\frac{dy}{dt} + 6y = \frac{du}{dt};\ \ y(0)=1,\ \frac{dy(0)}{dt}=-1$$

where u is the control input and y is the output.

(a) (10%) Find the transfer function of the system.

(b) (10%) Find the zero-input response $y(t)$.

(c) (10%) Choose, among the following types of control, ones that are likely to stabilize the system (**One or more answers may be correct. Answers without supporting explanation are not acceptable**) : (i) P control (ii) I control (iii) D control (iv) PID control (v) PD control (vi) PI control

(d) (10%) Choose, among the following types of control, ones that are likely to regulate the system output to a constant level 0.5 with zero steady state error (**One or more answers may be correct. Answers without supporting explanation are not acceptable**): (i) P control (ii) I control (iii) D control (iv) PID control (v) PD control (vi) PI control

PROBLEM 2 (30%)

An LCR circuit is shown in Fig. 1, with its control input to be u, and output $y = x_2$.

(a) (10%) Derive a state-space representation of this LCR circuit.

(b) (10%) Suppose $L = C = R = 1$ and $x_1(0) = x_2(0) = 1$. What is the output y in **steady state** if $u = \sin(t)$?

(c) (10%) Following 2(b), let the control be $u(t) = -k_1x_1(t) - k_2x_2(t)$. Choose the gains k_1 and k_2 such that the closed loop poles are –2 and –3.

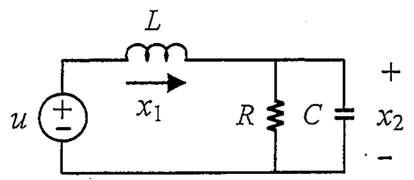

Fig. 1

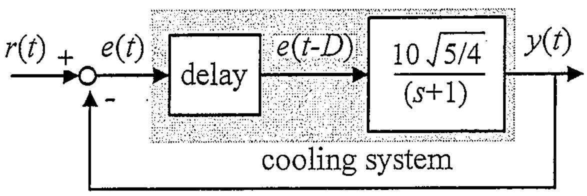

Fig. 2

PROBLEM 3 (20%)

Figure 2 shows a simple temperature control of a machine tool cooling system, where r is the setting temperature and y is the coolant temperature. There is a delay of D seconds present in the cooling system.

(a) (10%) Use **Laplace transform** to derive the transfer function from $e(t)$ to $e(t-D)$.

(b) (10%) Estimate the **maximum allowable delay** for the system to maintain at least 20 dB of gain margin, by using the approximation $\tan^{-1}\omega \approx \omega$, when $|\omega| << 1$.

PROBLEM 4 (10%)

After dozing off in the Monday morning class, your classmate asked you: "Integral control adds negative phase, thus decreasing the phase margin. Why would we ever want to use integral control?" Explain how you would answer him.