國立中山大學102學年度碩士暨碩士專班招生考試試題

科目名稱：離散數學【電機系碩士班丙組選考】　　　　　　題號：431003
※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）　　　　共2頁第1頁

考生請注意：1. 必須寫出作答過程或得到答案之理由，只寫答案不予計分。
2. 禁止在試題紙上作答。
3. 參考公式請見下頁。

1. Explain the following terms : [20%, 每小題5分]
   (a) Fundamental Theorem of Arithmetic
   (b) Homeomorphic graph
   (c) Equivalence relation
   (d) Four-color Theorem

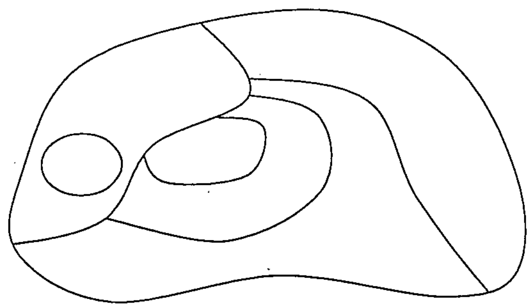
2. Given a number $x = 329313600$, please answer the following questions.
   (a) How many positive divisors does x have? [5%]
   (b) How many positive divisors of x that are divisible by 252? [5%]
   (c) Determine how many positive divisors of x are perfect squares? [5%]

3. (a) If an equivalence relation R on set $A = \{1,2,3,4,5\}$ induces the partition $A = \{1,3\} \cup \{2,4\} \cup \{5\}$, what is R? [5%]
   (b) Let R={(1,1),(1,2),(2,2),(2,4), (3,3) ,(3,4),(4,5),(5,5)}be a relation on A. What is the relation R^3 ? [5%]

4. What is the Ferrers graph? Use it to explain the statement "The number of partitions of an integer n into m summands is equal to the number of partitions of n into summands where m is the largest summand". [10%]

5. Use the generating function to find the number of integer solutions of the equation $x_1 + x_2 + x_3 + x_4 = 20$, where $-3 \le x_1, -3 \le x_2, -5 \le x_3 \le 5$, and $0 \le x_4$. [15%]

6. Find the number of colors needed to color the following map so that no two adjacent regions have the same color. [15%]



7. Find the number of permutation of the letters x, x, y, y, z, z so that no x appears in the first and second positions, no y appears in the third position and no z appears in the fifth and sixth positions. [15%]

Appendix:

1. $(1+x)^n = \binom{n}{0} + \binom{n}{1}x + \binom{n}{2}x^2 + \cdots + \binom{n}{n}x^n$
2. $(1+ax)^n = \binom{n}{0} + \binom{n}{1}ax + \binom{n}{2}a^2x^2 + \cdots + \binom{n}{n}a^nx^n$
3. $(1+x^m)^n = \binom{n}{0} + \binom{n}{1}x^m + \binom{n}{2}x^{2m} + \cdots + \binom{n}{n}x^{nm}$
4. $a(1-x^{n+1})/(1-x) = a + ax + ax^2 + \cdots + ax^n$
5. $1/(1-x) = 1 + x + x^2 + \cdots = \sum_{i=0}^{\infty} x^i$
6. $1/(1-ax) = 1 + ax + a^2x^2 + \cdots = \sum_{i=0}^{\infty} a^i x^i$
7. $1/(1+x)^n = 1 + (-1)\binom{n+1-1}{1}x + (-1)^2\binom{n+2-1}{2}x^2 + \cdots = \sum_{i=0}^{\infty}(-1)^i\binom{n+i-1}{i}x^i$
8. $1/(1-x)^n = 1 + (-1)\binom{n+1-1}{1}(-x) + (-1)^2\binom{n+2-1}{2}(-x)^2 + \cdots = \sum_{i=0}^{\infty}\binom{n+i-1}{i}x^i$
9. $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots = \sum_{i=0}^{\infty}\frac{x^i}{i!}$