國立中山大學102學年度碩士暨碩士專班招生考試試題

科目名稱：微積分【應數系碩士班乙組】　　題號：424004

※本科目依簡章規定「不可以」使用計算機　　

計算題：共7題，子題分數平均分配。答題時，每題都必須寫下題號與詳細步驟。

[1]. (18%) Evaluate the following limits.

(a) $\lim_{x\to\infty}\left(\frac{1-4x}{3-4x}\right)^{2-4x}$

(b) $\lim_{n\to\infty}\sum_{k=1}^{n}\cos^2\left(\frac{k\pi}{2n}\right)\frac{\pi}{n}$

[2]. (12%) Solve the differential equation $y'+y=e^x,\quad y(0)=2.$

[3]. (20%) Evaluate the following integrals

(a) $\int_{\sqrt{3}}^{\sqrt{6}}\frac{2}{x\sqrt{x^4-9}}dx$

(b) $\int_{-1}^{0}\frac{2x-2}{x^2+2x+2}dx$

[4]. (10%) Let $f(x,y)=e^{-(x^2+y^2)}$. Find the equation of the tangent plane at the point $(0,1)$.

[5]. (10%) Find the length of the curve $r=1+\cos\theta$ for $0\le\theta\le\pi$.

[6]. (14%) Evaluate the following iterated integral.

$$\int_0^2\int_0^{4-x^2}\frac{xe^{2y}}{4-y}dydx$$

[7]. (16%) Sketch the graph of $f(x)=\sqrt[3]{x^2(9-x)}$.
Determine the open interval on which the graph is increasing, decreasing, concave upward, or concave downward.

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