國立中山大學 102 學年度碩士暨碩士專班招生考試試題

科目名稱：線性代數【應數系碩士班乙組、丙組】　　題號：424002

※本科目依簡章規定「不可以」使用計算機　　共 1 頁第 1 頁

Do all the following problems. Show details of your work.

1 Let $A = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$. Find determinant of A and A^{-1} if it exists. (10%)

2. (a) Define $T : \mathbf{R}^4 \to \mathbf{R}^3$ by $T(s,t,x,y) = (s+x+y, s+t-x-y, 3s+t-y)$. Find bases of the kernel of T and the image space of T. (15%)

   (b) Define $T : \mathbf{R}^2 \to \mathbf{R}^2$ by $T(x,y) = (x-2y, x+y)$. Find the matrix representation of T relative to the ordered basis $B = \{(1,1), (-1,2)\}$ of $\mathbf{R}^2$. (10%)

3. Let A be an $n \times n$ matrix. Prove or disprove: If $\{\mathbf{v}_1, \ldots, \mathbf{v}_n\}$ and $\{A\mathbf{v}_1, \ldots, A\mathbf{v}_n\}$ are independent sets, then A is invertible. (10%)

4. Let T be a linear transformation on $\mathbf{R}^n$ such that $\text{rank}(T^2) = \text{rank}(T)$. Find $\text{Ker}(T) \cap \text{Im}(T)$. Justify your answer. (15%)

5. Let $A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 1 & -1 \\ 0 & 2 & 4 \end{bmatrix}$.

   (a) Find the characteristic polynomial of A. (5%)

   (b) Find the minimal polynomial of A. (5%)

   (c) Find a Jordan form for A. (5%)

   (d) Find a matrix P such that $P^{-1}AP$ is the Jordan form in (c). (10%)

6. Let $< \mathbf{u}, \mathbf{v} > = x_1y_1 + 4x_1y_2 + 4x_2y_1 - x_2y_2$, where $\mathbf{u} = (x_1, x_2)^T$ and $\mathbf{v} = (y_1, y_2)^T$.

   (a) Find a matrix A such that $< \mathbf{u}, \mathbf{v} > = (A\mathbf{u})^T\mathbf{v}$. (5%)

   (b) Does $< \cdot, \cdot >$ define an inner product on $\mathbf{R}^2$? Justify your answer. (10%)

End of Paper