

國立中山大學 102 學年度碩士暨碩士專班招生考試試題

科目名稱：基礎數學【應數系碩士班甲組】

題號：424001

※本科目依簡章規定「不可以」使用計算機

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答題時，每題須寫下題號與詳細步驟。請依題號順序作答，不會作答題目請寫下題號並留空白。

1. (10%) Evaluate $\lim_{x \rightarrow 0} 2^{x \cos(1/x)}$.

2. (10%) Evaluate $\int_0^{\pi/2} \frac{\cot x}{1 + \csc x} dx$.

3. (10%) Let $1 < a < b < \infty$. Prove that $\arctan b - \arctan a \leq (b - a)/2$.

4. (10%) Calculate the area of the region Ω enclosed by the curves

$$x^3 + y^3 + 3x^2y + 3xy^2 - 3x - y = 0 \quad \text{and} \quad x^2 + y^2 + 2xy - x + y = 0.$$

5. (20%) Let $f(x) = 2 + 3x + x^2 + 2x^3 + 3x^4 + x^5 + 2x^6 + 3x^7 + x^8 + \dots$. Find the interval of convergence for $f'(x)$.

6. (20%) Determine all possible values of a such that the matrix $\begin{pmatrix} 6 & -2 & 0 \\ -2 & a & -3 \\ 0 & -3 & 4 \end{pmatrix}$ is positive definite.

7. (20%) Let $A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 1 & 1 \end{pmatrix}$. Find matrices D and Q such that $A = QDQ^{-1}$, where D is the Jordan normal form of A and Q^{-1} is the inverse of Q .

