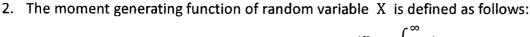
## 國立中央大學102學年度碩士班考試入學試題卷

所別:工業管理研究所碩士班 不分組(一般生) 科目:統計學 共 頁 第 頁

本科考試可使用計算器,廠牌、功能不拘

\*請在試卷答案卷(卡)內作答

1. A load of laundry contains 4 (2 pairs) tan socks and 4 grey socks. A color-blind person who cannot see the difference between tan and grey sorts the socks into pairs and gets 4 pairs of mismatched socks. Assume that pairing of socks is random. Let X be the number of mismatched pairs. Find the probability mass function of X (15 points)



 $M_X(t) = E[e^{tX}] = \int_{-\infty}^{\infty} e^{tx} f(x) dx$ 

, where f(x) is the pdf of X.

- (a) Find the moment generating function of X, where X is U(0,1) distributed. (5 points)
- (b) Find  $E[Y^2]$  from the moment generating function of Y, where Y = X + Z, X and Z are independent and U(0,1) distributed. (10 points)
- 3. Consider mutually independent  $\,\mathrm{U}(0,1)\,$  random variables  $\,X_1,X_2,\ldots$ . We are interested in the pdf of  $\,\mathrm{Z}_n,\,\,f_n(t),\,$  where  $\,Z_n=X_1X_2\ldots X_n.\,$  Let  $\,F_n(t)=\mathrm{P}(\mathrm{Z}_n\le t\,),\,$  where  $\,0< t< 1.$  By definition,  $\,F_1(t)=t\,$  and note that  $\,\mathrm{Z}_n=Z_{n-1}X_n.\,$ 
  - (a) Suppose that  $F_{n-1}$  is known. What is  $P(Z_n \le t | X_n = x)$  in terms of  $F_{n-1}$ ? (10 points)
  - (b) From (a), what is  $F_n(t)$  for  $0 \le t \le 1$  in terms of  $F_{n-1}$ ? (5 points)
  - (c) From (b), what is  $F_3(t)$ ? (5 points)
- 4. Assume that there is a relationship between random variable  $Y_i$  and the corresponding given value  $X_i$  as following:

$$Y_i = \beta X_i + \varepsilon_i$$
  $i = 1, 2, ..., n$ 

where  $\beta$  is an unknown parameter, and  $\varepsilon_i$  is independently and normally distributed with mean 0, and variance  $\sigma^2$ .

- (a) Find the least squares estimator  $\hat{\beta}$ , such that  $\sum_{i=1}^{n} [Y_i \hat{\beta}X_i]^2$  minimized. (10 points)
- (b) Show that whether or not the least squares estimator  $\hat{\beta}$  in (a) is the same as the maximum likelihood estimator. (5 points)
- (c) Given a particular value,  $x_p$ , what is the distribution of  $y_p$ , where  $y_p = \beta x_p + \varepsilon_i$ . (5 points)
- (d) Given a particular value,  $x_p$ , what is the distribution of  $\hat{y}_p$ , where  $\hat{y}_p = \hat{\beta} x_p$ . (5 points)
- (e) Construct an 100(1- $\alpha$ )% confidence interval of the expected value of  $y_p$ ,  $E(y_p)$ . (5 points)
- 5. A supplier ships parts to another company in lots of *N* parts, some of which could be defective. The receiving company uses an acceptance sampling plan (single-sampling plan) which defined by the sample size *n* without replacement and the acceptance number *c*. i.e. from a lot of size *N*, a random sample of *n* parts is inspected. If the number of defective items is less than or equal to *c*, the lot will be accepted.
  - (a) For a particular lot, assume the defective rate is p. Under the sampling plan (n=5, c=1), please find the probability of accepting the lot. (5 points)
  - (b) In (a), the accepting probability depends on the defective rate, p. If the acceptable quality level (AQL) is set at p=0.1, and the rejectable quality level (RQL) is p=0.2, the sampling plan is designed for the test H<sub>0</sub>: p=0.1 v.s. H<sub>A</sub>: p=0.2. Please find the type I error,  $\alpha$ , and the type II error,  $\beta$ , under the sampling plan (n=5, c=0). (5 points)
  - (c) If the desired type I error is  $\alpha$  =0.05, by using the sampling plan (n, c=0), what is the minimal required sampling size, n? What is the corresponding type II error,  $\beta$ ? (5 points)
  - (d) Furthermore, what is your suggested sampling plan to achieve  $\alpha$  and  $\beta$  are both less than 0.05 simultaneously. (5 points)