

系所組別： 交通管理科學系乙、丙組

考試科目： 微積分

考試日期：0224，節次：2

※ 考生請注意：本試題不可使用計算機

\*下列十題每題均為 10 分,總計 100 分.

1. Evaluate  $\int_0^1 \frac{x^3}{\sqrt{x^2+1}} dx$

2. Define  $F(x) = \int_0^{x^2} x\sqrt{1-t} dt$ , find  $F'(1)$

3. Find  $\lim_{x \rightarrow 0} \frac{\sin x - x}{x - \ln(1+x)}$

4. Find an equation of tangent line to the curve  $x = y + e^{\frac{x}{y}}$  at the point  $(0, -1)$

5. Find a power series representation for the function  $f(x) = x \cdot \ln(1+x^2)$ , and find the interval of convergence6. Find the volume of the solid bounded by the plane  $z = 0$  and the paraboloid  $z = 1 - x^2 - y^2$ 7. Let  $C(x)$  be the cost function of a catering service to cater  $x$  people. If its marginal cost can be modeled by  $\frac{dc}{dx} = \frac{5x}{\sqrt{x^2+6400}}$ , and when  $x = 150$ , the cost is \$1000. Find the cost function  $C(x)$ .8. The emergency stopping distance in meter for a truck of weight  $\omega$  tons traveling at  $v$  km/hr on a dry road is  $S = 0.002 \cdot \omega v^2$ For a truck that weight 4 tons and is usually driven at 90 km/hr, estimate the extra stopping distance if it has an extra half ton of load and is traveling 5 km/hr faster than usually. (Hint : Use  $\Delta S \approx dS$ , where  $dS$  is the total differentials)9. A company's profit is  $P = 300 \cdot x^{\frac{2}{3}} y^{\frac{1}{3}}$ , where  $x$  and  $y$  are respectively, the amounts spent on production and advertising. The company has a total of \$60,000 to spend. Use Lagrange multipliers to find the amount for production and advertising that maximize profit.10. A company sells two products whose demand functions are given by  $x_1 = 400 - 3P_1$ , and  $x_2 = 600 - 2.4P_2$ . So, the total revenue is given by  $R = x_1 P_1 + x_2 P_2$ . Estimate the average revenue if the price  $P_1$  varies between \$2 and \$5, as the price  $P_2$  varies between \$10 and \$20.