

系所組別： 統計學系

考試科目： 數學

考試日期： 0224，節次： 1

※ 考生請注意：本試題不可使用計算機

1. What is the area A of the region below the graph of $f(x) = 1/x$, $x \geq 1$? Suppose that this region is revolved about the axis. What is the volume V of the resulting solid? (10%)
2. Let r be a positive number. Show that $\lim_{n \rightarrow \infty} \frac{n!}{n^n} = 0$ and find $\lim_{n \rightarrow \infty} \frac{r^n}{n!}$. (10%)
3. Show that $\lim_{n \rightarrow \infty} \sqrt{n+1} - \sqrt{n} = 0$ and find $\lim_{n \rightarrow \infty} \sqrt{n^2 + n} - n$. (10%)
4. Find $\int \frac{e^x}{e^{2x} + 2e^x + 5} dx$ and $\int_0^1 \frac{e^x}{e^x - 2} dx$. (10%)
5. Find $H'(3)$ given that $H(x) = \frac{1}{x} \int_3^x [2t - 3H'(t)] dt$. (10%)
6. Assume that f is a continuous function and that (10%)
 $\int_0^x tf(t) dt = \sin x - x \cos x$. Determine $f(\pi/2)$ and find $f'(x)$.
7. Prove that, for any positive integer n : $\int_0^\pi \sin^2 nx dx = \frac{\pi}{2}$. (10%)
8. A is an m by n matrix of rank r . Suppose that there are right sides b for which $Ax = b$ has no solution. (15%)
 (a) What are all inequalities ($<$ or \leq) that must be true between m , n , and r ? (5%)
 (b) Does $A^T y = 0$ have solutions other than $y = 0$? Why? (4%)
 (c) Suppose $A^T y = d$ is solvable. Is the solution y unique? Why? (4%)
 (d) Suppose column 1 + column 3 + columns 4 = $\mathbf{0}$ in a 3 by 4 matrix with rank 3. What is the null space? (2%)
9. Find the product of all eigenvalues of $A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 3 & 2 & 1 & 0 \\ 4 & 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 4 & 3 & 2 & 1 \\ 0 & 7 & 6 & 5 \\ 0 & 0 & 9 & 8 \\ 0 & 0 & 0 & 10 \end{bmatrix}$. (5%)
10. Suppose $Ax = \lambda x$, $\lambda \neq 0$ and y is in the null space of A . Are x and y perpendicular? Are x and y independent? Is x in the row space of A ? Explain carefully. (10%)