1. (20%) Integral can be valued using numerical methods of integration. In this question, please use different numerical methods to approximate the value of a definite integral. Evaluate the integral $\int_1^2 \frac{1}{x^2} dx$, by dividing the interval $[1, 2]$ into 4 subintervals using

   (a) (5%) Riemann sum;

   (b) (5%) the Trapezoidal rule;

   (c) (5%) the Simpson's rule;

   (d) (5%) the exact value of the definite integral.

   Round your answer to four decimal places.

2. (10%) Suppose that f is differentiable on $(1, 4)$, continuous on $[1, 4]$, and $f(1) = 2$. Given that $2 \le f'(x) \le 3$ for all x in $(1, 4)$.

   (a) (5%) What is the least value that f can take on at 4?

   (b) (5%) What is the greatest value that f can take on at 4?

3. (20%) Find the following integrals.

   (a) (10%) $\int_1^4 \frac{5^{\sqrt{x}}}{\sqrt{x}} dx$

   (b) (10%) $\int \frac{x-11}{x^2+3x-4} dx$

4. (20%) Please determine the convergence or divergence of the following series:

   (a) (10%) $\sum_{k=1}^{\infty} k e^{-3k^2}$

   (b) (10%) $\sum_{n=1}^{\infty} \frac{n^2}{n!}$

5. (15%) Please solve the following differential equation.

$$x \frac{dy}{dx} + 2y = 1 - \frac{1}{x}, x > 0.$$

6. (15%) Please evaluate the integral $\int_0^1 \int_y^1 x^2 e^{xy} dx dy$.