考試科目	基礎數學	所別	統計學系	考試時間	2 月 23 日(六) 第 一 節

Part I: 微積分 (填充題)

(1) (5%) $\lim_{x\to\infty} x-\sqrt{x^2+7} =$ _____.

(2) (5%) Assume $\lim_{x\to -1^-} f(x)$ exists and $\frac{x^2+x-2}{x+3} \le \frac{f(x)}{x^2} \le \frac{x^2+2x-1}{x+3}$, then $\lim_{x\to -1^-} f(x) =$ _____.

(3) (5%) For what values of x is the function $f(x)=\sqrt{x^2-2x}$ continuous? _____.

(4) (5%) Let $y=\log(4+\cos x)$, $\frac{dy}{dx}=$ _____.

(5) (5%) Assume that y is a function of x such that $x^3+y^3=4$, then $\frac{dy}{dx}=$ _____.

(6) (5%) Let $f(x)=\int_x^{2x} \sin(t^2)\,dt$, then $f'(x) =$ _____.

(7) (5%) $\lim_{n\to\infty}\sum_{i=1}^{n}\left(\frac{10}{n}+\frac{9i}{n^2}\right)=$ _____.

(8) (5%) $\int \frac{x}{x^2+4}\,dx =$ _____.

(9) (5%) Write down the first three nonzero terms of Taylor series for $f(x)=\log\cos x$ at $x=0$.

_____.

(10) (5%) Let $f(x)=2x^3-9x^2+12x-3$, the maximum value of $f(x)=$ _____.

Part II. 線性代數 (填充題)

(11) (5%) Consider the linear system

$$\begin{aligned} x+ \quad & y= 4 \\ x+(a^2-15)y &= a \end{aligned}$$

What are the value(s) of a so that the linear system has a unique solution: _____.

(12) (5%) For what value(s) of λ is the set of vectors $\{(\lambda^2-5,\ 1,\ 0),\ (2,-2,\ 3),\ (2,\ 3,-3)\}$ linearly dependent ? _____

備註	試題隨卷繳交

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(13) (5%) Let matrix $A=\begin{bmatrix} 1 & 2 & 7 & 5 \\ -2 & -1 & -8 & -7 \\ -1 & 3 & 3 & 0 \end{bmatrix}$, the rank of A = _____.

(14) (5%) Let $A=\begin{bmatrix} 4 & -4 & 2 \\ 2 & -2 & 2 \\ 0 & 0 & 1 \end{bmatrix}$, what is the eigenvector $\mathbf{v}$ corresponding to the eigenvalue $\lambda = 0$?

$\mathbf{v}$ = _____.

(15) (5%) Let $A=\begin{bmatrix} 0 & -2 & 1 \\ 1 & 3 & -1 \\ 0 & 0 & 1 \end{bmatrix}$, find a diagonal matrix B similar to A. B = _____

(16) (5%) Let $A=\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix}$, is A an orthogonal matrix ? (Yes/No) _____

(17) (5%) Continue with problem (16), $\det(A^{-1})$ = _____.

(18) (5%) Let $L: R^3 \to R^3$ be defined by

$$L(x,y,z)=(x-y,x+2y,z).$$

Find a basis for the kernel of L. _____

(19) (5%) Continue with problem (18), find a basis for the range of L. _____

(20) (5%) Let V be the vector space consisting of all functions of the form

$$ae^{2x}\cos x + be^{2x}\sin x.$$

Consider a linear transformation $L: V \to V$ such that $L(f)=f'+f$.

Find the matrix representing L with respect to the basis $\{e^{2x}\cos x,\ e^{2x}\sin x\}$. _____

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