

## Probability & Statistics 2008 (100%)

1. [20%] Let  $X$  denote the number of times a certain numerical control machine will malfunction: 1, 2, or 3 times on any given day. Let  $Y$  denote the number of times a technician is called on an emergency call. Their joint probability distribution is given as:

$f(x, y)$	$x$	1	2	3
$y$	1	0.05	0.05	0.1
	2	0.05	0.1	0.35
	3	0	0.2	0.1

- (a) Evaluate the marginal distribution of  $X$ .  
 (b) Evaluate the marginal distribution of  $Y$ .  
 (c) Find  $P(Y = 3 | X = 2)$ .  
 (d) Find the covariance of  $X$  and  $Y$ .
2. [15%] At "busy time" a telephone exchange is very near capacity, so callers have difficulty placing their calls. It may be of interest to know the number of attempts necessary in order to gain a connection. Suppose that let  $p = 0.05$  be the probability of a connection during busy time. What is the probability that 5 attempts are necessary for a successful call?
3. [20%] Prove that the mean and variance of the gamma distribution with the following density function are  $\mu = \alpha\beta$ ;  $\sigma^2 = \alpha\beta^2$ .

$$f(x) = \begin{cases} \frac{1}{\beta^\alpha \Gamma(\alpha)} x^{\alpha-1} e^{-x/\beta}, & x > 0 \\ 0, & \text{elsewhere} \end{cases}$$

where  $\alpha > 0$  and  $\beta > 0$ .

4. [10%] A foreign student club lists as its members 2 Canadians, 3 Japanese, 5 Italians, and 2 Germans. If a committee of 4 is selected at random, find the probability that.
- (a) all nationalities are represented;  
 (b) all nationalities except the Italians are represented.
5. [10%] Please describe Central Limit Theorem.
6. [15%] A random sample of 100 recorded deaths in the United States during the past year showed an average life span of 71.8 years. Assuming a population standard deviation of 8.9 years, does this seem to indicate that the mean life span today is greater than 70 years? Use a 0.05 level of significance.

This table indicates the area under the standard normal curve corresponding to  $P(Z < z)$

$z$	.00	.01	.02	.03	.04	.05
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678

7. [10%] Please draw normal curves with two cases:  $\mu_1 = \mu_2$ ,  $\sigma_1 < \sigma_2$ ; and  $\mu_1 < \mu_2$ ,  $\sigma_1 < \sigma_2$ .