

本試題是否可以使用計算機：☐可使用，☒不可使用（請命題老師勾選）

考試日期：0301，節次：3

1. Please find the derivatives of the following functions. (18 points)

(a) $y = x^x$ (b) $y = x^2 e^x$ (c) $y = \log(x + \sqrt{1+x^2})$

2. Please evaluate the limit of the following functions. (7 + 8 = 15 points)

(a) $\lim_{n \rightarrow \infty} \frac{(n-1)(3n+1)}{n^2+1}$ (b) $\lim_{n \rightarrow \infty} \sum_{k=1}^n \left\{ \frac{1}{n} \left(\frac{k}{n} \right)^4 \right\}$

3. Please answer the following questions.

(a) Please find the tangent plane of $z = f(x, y) = x^2 y$ at $(x, y, z) = (1, 1, 1)$. (5 points)(b) Please obtain minimum value of $z = 6x^2 + 9y^2 - 4xy - 8x + 6y + 3$ (8 points)(c) Obtain the length of the arc: $y = x^2$ ($0 \leq x \leq 1$) (8 points)

4. Suppose there is a point P on a curve. Consider a circle that is tangent to the curve at P, and crosses with the curve at another point Q. When the point Q comes close to point P, the limit of the radius of the circle is called "radius of curvature".

(a) Express an equation for the circle which is tangent to $y = \cos x$ at $P(0, 1)$, whose radius = r . (3 points)(b) Please obtain radius of curvature for $y = \cos x$ at $P(0, 1)$. (8 points)

5. Please find the integrals of the following functions. (20 points)

(a) $\int \frac{x^4}{x^3 - x^2 - x + 1} dx$ (b) $\int_0^{\pi} x^2 \sin x dx$ (c) $\int_1^e (\log x)^2 dx$ (d) $\int_1^2 \frac{1 - \log x}{x^2} dx$

6. Obtain the volume of the region surrounded by the following three surfaces: 1) a surface that could be obtained by rotating $y = e^x$ around the x axis, 2) $y = 1$, and 3) $x = \log_e \sqrt{2}$.(One of the fractions, which is $y \geq 1$). (15 points)