

1. (20%)

A system undergoing a thermodynamic cycle receives  $Q_H$  at temperature  $T'_H$  and discharges  $Q_C$  at temperature  $T'_C$ . There are no other heat transfers.

(a) Show that the net work developed per cycle is given by

$$W_{\text{cycle}} = Q_H \left( 1 - \frac{T'_C}{T'_H} \right) - T'_C \sigma$$

where  $\sigma$  is the amount of entropy produced per cycle owing to irreversibilities within the system.

(b) If the heat transfers  $Q_H$  and  $Q_C$  are with hot and cold reservoirs, respectively, what is the relationship of  $T'_H$  to the temperature of the hot reservoir  $T_H$  and the relationship of  $T'_C$  to the temperature of the cold reservoir  $T_C$ ?

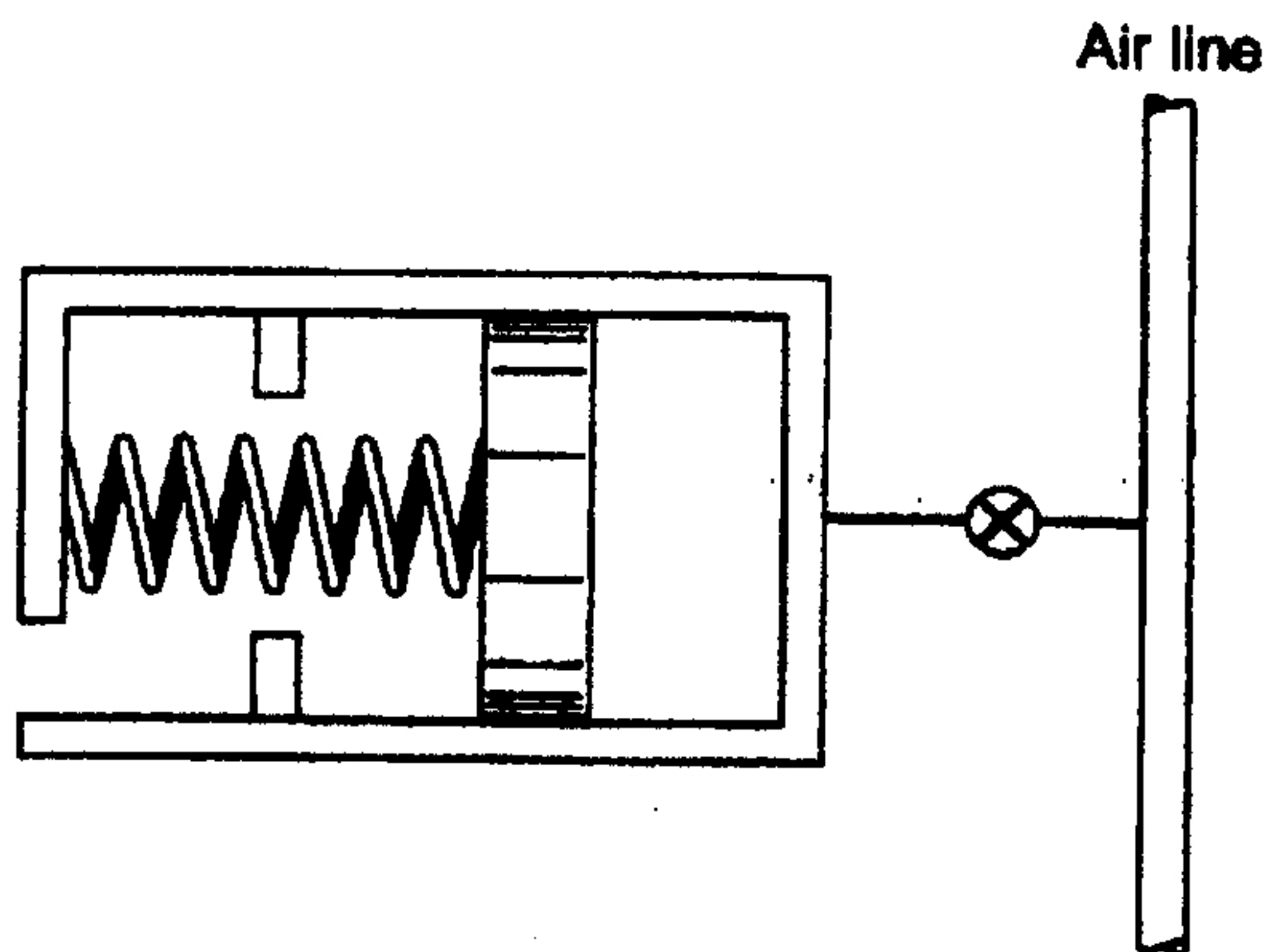
(c) Obtain an expression for  $W_{\text{cycle}}$  if there are (i) no internal irreversibilities; (ii) no internal or external irreversibilities.

(背面仍有題目,請繼續作答)

2. (30%)

A frictionless piston/cylinder is loaded with a linear spring, spring constant  $100 \text{ kN/m}$ , and the piston cross-sectional area is  $0.1 \text{ m}^2$ . The cylinder initial volume of  $20 \text{ L}$  contains air at  $200 \text{ kPa}$ . The ambient temperature is  $10^\circ\text{C}$ . The cylinder has a set of stops that prevents its volume from exceeding  $50 \text{ L}$ . A valve connects to a line flowing air at  $800 \text{ kPa}$ ,  $50^\circ\text{C}$ , as shown in the following figure. The valve is now opened, allowing air to flow in until the cylinder pressure reaches  $800 \text{ kPa}$ , at which point the temperature inside the cylinder is  $80^\circ\text{C}$ . The valve is then closed and the process ends. Assume air is an ideal gas, with constant specific heat,  $C_p = 1.004 \text{ kJ/kg}\cdot\text{K}$ ,  $C_v = 0.717 \text{ kJ/kg}\cdot\text{K}$ , and  $R = 0.287 \text{ kJ/kg}\cdot\text{K}$ .

- Taking the inside of the cylinder as a control volume, calculate the work and heat transfer during the process.
- Show that this process does not violate the second law.
- Plot the process in a P-V diagram and clearly mark each state.



3. Nitrogen ( $N_2$ ) is the working fluid of a Stirling cycle with a compression ratio of nine. At the beginning of the isothermal compression, the temperature, pressure, and volume are 300K, 1 bar, and  $0.008 \text{ m}^3$ , respectively. The temperature during the isothermal compression is 1000 K. (a) Plot the  $p$ - $v$  diagram of the cycle. Determine (b) the net work, in kJ, and (c) the mean effective pressure, in bar. (d) Why is the thermal efficiency of the Stirling cycle given by the same expression as for the Carnot cycle. (20 %)

4. If a pure substance undergoes an infinitesimal reversible process between two equilibrium states, the change of internal energy is

$$du = Tds - Pdv.$$

- (a) Show that

$$\left(\frac{\partial u}{\partial v}\right)_T = T\left(\frac{\partial P}{\partial T}\right)_v - P.$$

- (b) Using the virial expansion

$$Pv = RT\left(1 + \frac{B}{v} + \frac{C}{v^2} + \dots\right),$$

derive the expression of  $\left(\frac{\partial u}{\partial v}\right)_T$  in terms of  $v$  and  $T$ .

- (c) Using the same expansion, derive the expression of  $\left(\frac{\partial u}{\partial P}\right)_T$  in terms of  $v$  and  $T$ .

(30 %)