

一、選擇題 50 分(每題五分)

1. $\sin^{-1} x + \cos^{-1} x =$ (a) $\frac{\sqrt{\pi}}{2}$ (b) π (c) $\frac{2}{\pi}$ (d) $\frac{\pi}{2}$
2. The area of the curve $y = \sqrt{1-x^2}$ from $x = -1$ to $x = 1$ is rotated about the X-axis. Find the resulting surface area (a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$ (c) 2π (d) 4π
3. $\int_2^{2 \cosh x} \frac{dt}{\sqrt{t^2 - 4}} =$ (a) $\sinh x$ (b) $\sin^{-1} x$ (c) x (d) x^2
4. If $x > 0$, $\lim_{n \rightarrow \infty} \sqrt[n]{x} =$ (a) ∞ (b) 1 (c) 0 (d) x
5. $\lim_{n \rightarrow \infty} (\sqrt{n^2 + n} - n) =$ (a) ∞ (b) $-\infty$ (c) 0 (d) $\frac{1}{2}$
6. $\int \frac{x^5 - x^4 - 2x^3 + 4x^2 - 15x + 5}{(x^2 + 1)^2(x^2 + 4)} dx =$
 (a) $\frac{1}{2} \ln|x^2 + 4| - \frac{3}{2} \tan^{-1} \frac{x}{2} + 2 \tan^{-1} x + \frac{2}{x^2 + 1} + C$
 (b) $\frac{1}{2} \ln|x^2 + 4| + \frac{3}{2} \tan^{-1} 2x + 2 \tan^{-1} x + \frac{2}{x^2 + 1} + C$
 (c) $\frac{1}{2} \ln|x^2 + 4| + \frac{3}{2} \tan^{-1} \frac{x}{2} + 2 \tan^{-1} x^2 + \frac{2}{x^2 + 1} + C$
 (d) $\frac{1}{2} \ln|x^2 + 4| - \frac{3}{2} \tan^{-1} 2x + 2 \tan^{-1} x^2 + \frac{2}{x^2 + 1} + C$
7. Evaluate $\int \sin(\ln(x)) dx =$ (a) $\frac{x}{2} [\sin(\ln x) + \cos(\ln x)] + C$
 (b) $\frac{x}{2} [\sin(\ln x) - \cos(\ln x)] + C$
 (c) $\frac{1}{2} [\sin(\ln x) + \cos(\ln x)] + C$
 (d) $\frac{1}{2} [\sin(\ln x) - \cos(\ln x)] + C$
8. Find the length of the arc of the cardioid $r = 1 - \cos \theta$ from $\theta = 0$ to $\theta = 2\pi$
 (a) 2 (b) 4 (c) 8 (d) 12
9. Find $\lim_{x \rightarrow 1^+} \left(\frac{1}{x-1} - \frac{1}{\ln x} \right) =$ (a) 0 (b) ∞ (c) $\frac{1}{2}$ (d) $-\frac{1}{2}$

(背面仍有題目, 請繼續作答)

10. Evaluate $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n+1} + \sqrt{n}}$ (a) 0 (b) -1 (c) $\frac{1}{2}$ (d) ∞

二、非選擇題 50 分

1. (15%) Evaluate the following functions:

a. $\lim_{x \rightarrow 0} (\cos x)^{1/x^2}$ (5%)

b. $\int \sinh^{-1} x \, dx$ (5%)

c. Find the derivatives of x^{x^x} (5%)

2. (10%) Prove that $x \geq -1$, and n is a positive integer, then

$$(1+x)^n \geq 1+nx$$

3. (10%) Show that

$$\int_0^{\infty} e^{-x^2} \, dx = \frac{1}{2} \sqrt{\pi}$$

4. (5%) Prove that

$$\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$$
 converges, and find its sum.

5. (10%) Prove that

$$\int_0^{\infty} e^{-x} x^n \, dx = n! \quad (n \in \mathbb{N}), \quad N \text{ is positive integer.}$$