

本科目不能使用計算機

第一部份、填充題（不需計算過程）：每格 6 分，共 60 分，

請以 (A), (B), (C), ... 依序標示清楚。

- Find  $\iint_R \cos(x-y) \sin(x+y) dx dy = \underline{\text{(A)}}$ , where  $R$  is the triangle with vertices  $(0, 0)$ ,  $(\pi, -\pi)$ , and  $(\pi, \pi)$ .
- Find  $\int \frac{1}{\sqrt{1-6x-x^2}} dx = \underline{\text{(B)}}$ .
- Find the minimum distance  $\underline{\text{(C)}}$  between the origin and the surface  $z^2 = x^2 y + 4$ .
- For what values  $\underline{\text{(D)}}$  of  $p$  does the integral  $\int_0^1 \frac{1}{x^p} dx$  diverge?
- Find the first three nonzero terms  $\underline{\text{(E)}}$  of the Maclaurin series for  $\frac{1}{1+\sin x}$ .
- Find  $\lim_{x \rightarrow \pi/2} (\sin x)^{\tan x} = \underline{\text{(F)}}$ .
- Find the length  $\underline{\text{(G)}}$  of  $y = \int_{\pi/6}^x \sqrt{64 \sin^2 u \cos^4 u - 1} du$ ,  $\frac{\pi}{6} \leq x \leq \frac{\pi}{3}$ .
- Find  $f(x) = \underline{\text{(H)}}$  if  $\int_0^{x^2} f(t) dt = \frac{1}{3} x^3$ .
- Find the equation  $\underline{\text{(I)}}$  of the normal line (line perpendicular to the tangent line) to the curve  $8(x^2 + y^2)^2 = 100(x^2 - y^2)$  at  $(3, 1)$ .
- The Mean Value Theorem for Derivatives says that if  $f$  is continuous on  $[a, b]$  and differentiable on  $(a, b)$ , then there is a point  $c$  in  $(a, b)$  such that  $\underline{\text{(J)}}$ .

第二部份、計算題（需詳細計算過程，否則不予計分），

每題 10 分，共 40 分

- A cylindrical can (圓柱筒) is to be made to hold 1L of oil. Find the dimensions that will minimize the cost of the metal to manufacture the can.
- Find the volume of a pyramid (金字塔) whose base is a square with side  $L$  and whose height is  $h$ .
- For what values of  $x$  is the series  $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{2^{2n} (n!)^2}$  convergent?
- Sketch the graph of  $y = f(x) = \frac{2x^2}{x^2 - 1}$ .