國立臺北大學九十七學年度碩士班招生考試試題

系（所）別：經濟學系

科　　目：統計學

☑可　☐不可使用計算機

I. Let $X_1, X_2, \ldots, X_{n_X}$ be a random sample from N(μ_X, σ_X^2); $Y_1, Y_2, \ldots, Y_{n_Y}$ be a random sample from N(μ_Y, σ_Y^2), where X and Y are independent. For each of the following statistics, write down the associated probability distributions and the corresponding parameters.

(a) $\sum_{i=1}^{n_X}(X_i-\mu_X)^2/\sigma_X^2+\sum_{i=1}^{n_Y}(Y_i-\mu_Y)^2/\sigma_Y^2$ (5%)

(b) $(\overline{X}-\overline{Y})-(\mu_X-\mu_Y)$ (5%)

(c) $n_X(\overline{X}-\mu_X)^2/S_X^2$, where $S_X^2=\sum_{i=1}^{n_X}(X_i-\overline{X})^2/(n_X-1)$ (5%)

II. Answer the following questions.

(a) For testing H_0: $\mu_X \le \mu_Y$ vs. H_1: $\mu_X > \mu_Y$, an analyst uses $\dfrac{\overline{X}-\overline{Y}}{S_p\sqrt{\dfrac{1}{n_X}+\dfrac{1}{n_Y}}}$ as the test statistic,

where $S_p^2=\dfrac{\sum_{i=1}^{n_X}(X_i-\overline{X})^2+\sum_{i=1}^{n_Y}(Y_i-\overline{Y})^2}{n_X+n_Y-2}$, and t_{α,n_X+n_Y-2} as the critical value. What assumptions are required for this test procedure? (5%)

(b) Comment on the following statement: In a hypothesis test, p-value increases as α (probability of type I error) decreases. (5%)

(c) Comment on the following statement: If the correlation coefficient of two variables is zero, then the two variables are not related. (5%)

III. The random variable W given θ is distributed with p.d.f. $f(w) = (1/\theta)e^{-(w-5)/\theta}$, $w > 5$.

(a) To test H_0: $\theta \ge \theta_0$ vs. H_1: $\theta < \theta_0$, using the Maximum Likelihood estimator for θ as the test statistic, derive the corresponding decision rule by using normal approximation. (12%)

(b) If θ is a random variable uniformly distributed in-between 2 and 4, find E(W). (8%)

IV. Fill in the following blanks.(50%)

1. Let $E(Y|X_1, X_2, X_3, X_4)=\beta_0+\beta_1X_1+\beta_2X_2+\beta_3X_3+\beta_4X_4$. The following regressions were run using 22 observations.

$$\underset{(1.800)}{\hat{Y}=2.940}-\underset{(0.879)}{2.200X_1}+\underset{(0.721)}{1.925X_2}+\underset{(0.221)}{0.163X_3}+\underset{(0.420)}{0.271X_4},\qquad R^2=0.6947 \qquad (1)$$

$$\underset{(2.000)}{\hat{Y}+X_1=2.837}-\underset{(0.987)}{1.481X_1}+\underset{(0.690)}{1.181X_2}+\underset{(0.134)}{0.186X_3}+\underset{(0.370)}{0.257X_4},\qquad R^2=0.6788 \qquad (2)$$

$$\underset{(0.820)}{\hat{Y}+X_1=-0.738}-\underset{(0.155)}{0.199X_3}+\underset{(0.165)}{0.261X_4},\qquad R^2=0.4277 \qquad (3)$$

Calculate the F statistic to test the null hypothesis of $H_0: \beta_1=-1$ and $\beta_2=0$. ____(1)____

2. Suppose regression model $Y_i=\beta_0+\beta_1X_i+u_i$ has $Var(u_i|X)=\sigma_i^2$. Thus, we divide by σ_i to correct the heteroscedasticity problem. Then, the regression model becomes $\dfrac{Y_i}{\sigma_i}=\beta_0\dfrac{1}{\sigma_i}+\beta_1\dfrac{X_i}{\sigma_i}+\dfrac{u_i}{\sigma_i}$. Write the OLS normal equations, which can be used to solve estimators $\tilde{\beta}_0$ and $\tilde{\beta}_1$. ____(2)____ (Two answers for this blank)

試題隨卷繳交　　　　接背面

國立臺北大學九十七學年度碩士班招生考試試題

系（所）別：經濟學系
科　　目：統計學

☑可　☐不可使用計算機

3. Suppose we run a regression of $w = \alpha + \beta m + u$, where w is hourly wage measured in dollars, m is a gender dummy taking the value 1 for a male and 0 for a female. It is known that the data set has observations on 284 males and 244 females. The sample mean and standard deviation of the dependent variable are 9.05 and 5.14, respectively. The following partial results are displayed:

$$\underset{(0.322)}{\hat{w} = 7.87} \quad \underset{(\ ?\ )}{+2.189m}, \quad s = 5.03, \qquad R^2 = ?$$

What is the value of the β estimate's standard error? ___(3)___
And what is the value of R^2? ___(4)___

4. Suppose that we have estimated the parameters of the multiple regression model:

$$Y_t = \beta_1 + \beta_2 X_{2t} + \beta_3 X_{3t} + u_t$$

by ordinary least squares (OLS) method. Denote the estimated residuals by $\hat{u}_t$ and the predicted values by $\hat{Y}_t$, $t = 1, \ldots, T$. If we regress Y on a constant and $\hat{Y}$, what are the estimated intercept and slope coefficients ($\hat{\alpha}$ and $\hat{\beta}$)? ___(5)___ (Two answers for this blank) What is the relationship between the R^2 of this regression (R^2_{new}) and the R^2 of the original Regression (R^2_{old})? ___(6)___

5. Consider the models $Y_t = \beta X_t + \varepsilon_t$, where $\varepsilon_t = u_t - \lambda u_{t-1}$. $E(u_t) = 0$, $E(u_t^2) = \sigma_u^2$, and $Cov(u_t, u_s) = 0$ if $t \neq s$. The stationarity condition is assumed, and $Cov(u_t, \varepsilon_s) = 0$ if $t > s$. Suppose the model is transformed into a new one by taking the first difference, i.e., $Y_t - Y_{t-1} = \beta(X_t - X_{t-1}) + v_t$, where $v_t = \varepsilon_t - \varepsilon_{t-1}$. Calculate the first-order autocorrelation coefficients for the original model's ε_t and the new model's v_t. ___(7)___ (Two answers for this blank) In order to obtain efficient estimate for the new model's parameter, what kind of estimation method should you use? ___(8)___

6. The following system models supply and demand of gasoline across 72 counties in a Province:

   Demand: $q = \alpha_1 + \alpha_2 p + \alpha_3 income + \alpha_4 cars + u_1$
   Supply: $q = \alpha_5 + \alpha_6 p + u_2$.

   Here q (quantity) and p (price) are the endogenous variables, while *income* (average household income) and *cars* (number of vehicles) are the exogenous variables. u_1 and u_2 have zero expectations and are independent of the exogenous variables. Two reduced form regressions were also obtained:

$$\hat{q} = 6 + 10 income + 5 cars \quad \text{and} \quad \hat{p} = 1 + 2 income + cars.$$

Find estimates of the supply function coefficients, $\hat{\alpha}_5$ and $\hat{\alpha}_6$. ___(9)___ (Two answers for this blank) And how would you estimate the demand function? ___(10)___ (A simple explanation is needed for this blank)

試題隨卷繳交