國立臺北科技大學九十七學年度碩士班招生考試

系所組別：2320 資訊工程系碩士班乙組

第一節　工程數學　試題

填准考證號碼

第一頁　共一頁

注意事項：

1. 本試題共九題，配分共 100 分。
2. 請標明大題、子題編號作答，不必抄題。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

一、(20%, 5% each) Give the definition for each of the following underlined terms.

1. A linear transformation T from the domain D to the co-domain C.
2. A basis B for a vector space V.
3. A discrete random variable X for a random experiment with the sample space S.
4. A normal (Gaussian) random variable X.

二、(10%) Let

$$A=\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & -2 \\ -1 & 0 & 1 \end{bmatrix} \text{ and } C=\begin{bmatrix} 2 & 1 \\ -1 & 0 \\ 0 & -1 \end{bmatrix}.$$

1. Calculate AC. (5%)
2. Calculate A^{-1} using row reductions. (5%)

三、(10%) For each of the following two matrices A and B, determine its *invertibility* and *diagonalizability* respectively. Use as few computations as possible.

$$A=\begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},\quad B=\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 3 \\ 0 & 0 & 0 \end{bmatrix}.$$

四、(10%) Let A be an $m\times n$ matrix. Prove that the orthogonal complement of the row space of A is the nullspace of A, i.e., $(\text{Row } A)^{\perp} = \text{Nul } A$. (*Hint*: consider $A\mathbf{x} = \mathbf{0}$.)

五、(10%) Suppose that A is an $n\times n$ real-valued symmetric matrix. Show that all the eigenvalues of A are real. (*Hint*: consider the quadratic form $\bar{\mathbf{x}}^T A\mathbf{x}$, where $\bar{\mathbf{x}}^T$ denotes the conjugate transpose of the vector $\mathbf{x}$.)

六、(10%) A box contains 5 red balls and 3 blue balls. Suppose that two balls are selected from the box at random *without replacement*. Let A be the event that the first ball is red and B be the event that the second ball is red. Find $P(A)$, $P(B|A)$, and $P(B)$. Are A and B independent?

七、(10%) A dice is rolled 10 times. Suppose for each time $P(1) = P(2) = 1/4$ and $P(3) = P(4) = P(5) = P(6) = 1/8$. What is the expected value of the sum of the squares of the outcomes?

八、(10%) Suppose that earthquakes occur in a certain region of Taiwan, in accordance with a Poisson process, at a rate of four per year.

1. What is the probability of no earthquakes in a year? (5%)
2. What is the probability that there will be at least one earthquake in the next six months (i.e., in half a year)? (5%)

九、(10%) Let $p_{XY}(x, y) = P\{X=x, Y=y\}$ denote the joint probability mass function of two random variables X and Y. Suppose that $p_{XY}(0, 0) = 1/2$, $p_{XY}(-1, 0) = 1/12$, $p_{XY}(1, 0) = 1/6$, and $p_{XY}(0, 1) = 1/4$. Calculate the marginal probability mass function of X and Y, respectively. Are X and Y independent? Are X and Y uncorrelated? Explain.

CS4-1