國立台灣科技大學九十七學年度碩士班招生試題

系所組別：財務金融研究所碩士班

科　　目：統計學

※總分100分

1. A sociologist hypothesizes that later-born children maintain closer relationships with their families than do their older brothers and sisters. To check her hypothesis, she selects a random sample of 20 college students from each of three groups. Group 1 consists of students who are oldest children, group 2 of students who have both older and younger siblings, and group 3 of students who are youngest children. She then obtains information about the number of letters written home by each student over a period of 6 months at college. The results are as follows.

Letter frequency	Family Position		
	Oldest	Middle	Youngest
At least twice per month	7	14	12
Less than twice per month	13	6	8

Calculate the appropriate statistic for this data under the hypothesis that family position has no effect on letter writing frequency and show how to justify this hypothesis. (10 points)

2. Experience has shown that while walking in a certain park, the time X, in minutes, between seeing two people smoking has a density function of the form as follows:

$$f(x)=\begin{cases}\lambda x e^{-x} & x>0\\ 0 & \text{otherwise.}\end{cases}$$

(a) Calculate the value of λ. (5 points)

(b) Find the probability distribution function of X. (5 points)

(c) What is the probability that Jack, who just seen a person smoking, will see another person smoking in 2 to 5 minutes? In at least 7 minutes? (10 points)

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3. Consider the following data and models:

obs.	x_i	y_i	x_i^2	y_i^2	x_iy_i	$(x_i-\bar{x})^2$	$(y_i-\bar{y})^2$	$(x_i-\bar{x})(y_i-\bar{y})$
1	1	-3	1	9	-3	4	1.44	-2.4
2	2	8	4	64	16	1	148.84	-12.2
3	2	1	4	1	2	1	27.04	-5.2
4	2	2	4	4	4	1	38.44	-6.2
5	3	0	9	0	0	0	17.64	0.0
6	3	-7	9	49	-21	0	7.84	0.0
7	4	-9	16	81	-36	1	23.04	-4.8
8	4	-15	16	225	-60	1	116.64	-10.8
9	4	-2	16	4	-8	1	4.84	2.2
10	5	-17	25	289	-85	4	163.84	-25.6
Sum	30	-42	104	726	-191	14	549.60	-65.0

Model 1: $y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$

Model 2: $y_i = \beta_0 + \varepsilon_i$

Model 3: $y_i = \beta_1 x_i + \varepsilon_i$

(a) Calculate the least squares estimates (i.e., $\hat{\beta}_0$ and $\hat{\beta}_1$) and corresponding standard error (i.e., $S_{\hat{\beta}_0}$ and $S_{\hat{\beta}_1}$) for the β_0 and β_1 coefficients of the Model 1. (10 points)

(b) Calculate the least squares estimates (i.e., $\hat{\beta}_0$) and corresponding standard error (i.e., $S_{\hat{\beta}_0}$) for the β_0 coefficient of the Model 2. (5 points)

(c) Calculate the least squares estimates (i.e., $\hat{\beta}_1$) and corresponding standard error (i.e., $S_{\hat{\beta}_1}$) for the β_1 coefficient of the Model 3. (5 points)

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4. (30%)Supposed we took a pair of random sample (X_1, X_2) with replacement from population $f(x)$

X	0	1	2	3
$f(x)$	0.2	0.3	0.4	0.1

(1)Construct the sampling distribution of $\overline{X}$.

(2)Find E($\overline{X}$).

(3)Var($\overline{X}$).

(4)Find P($\overline{X} > 1$).

(5)P($X_1 = 3, X_2 \geq 2$).

(6)Cov(X_1, X_2).

5. (20%)Two hypotheses concerning the probability density function of a random variable X are

$$H_0: \quad f(x) = \begin{cases} \frac{1}{4}(x+1) & 0 < x < 2 \\ 0 & otherwise \end{cases}$$

$$H_1: \quad f(x) = \begin{cases} \frac{1}{4}x^3 & 0 < x < 2 \\ 0 & otherwise \end{cases}$$

(1) Sketch the *p.d.f.* in each case.

The following test procedure is decided upon. A single observation of X is made and if X is less than a particular value k, where $0 < k < 2$, then H_0 is accepted, otherwise H_1 is accepted.

(2) Find k if P(Type I error) is 0.1 .

(3) With this value of k, find P(Type II error).