

國立台灣科技大學九十七學年度碩士班招生試題

系組別： 化學工程系碩士班

目： 工程數學

總分 100 分，請依序作答，並詳列計算過程。

1. Solve the following differential equation for $x > 0$, (15%)

$$x^2 \frac{d^2 y}{dx^2} + 10x \frac{dy}{dx} + 20y = 4 \ln(x) - x$$

2. Determine Laplace transform of $e^{-2t} \int_0^t e^{2\omega} \cos(3\omega) d\omega$

(a) using the shifting in s variable theorem.

(b) using the convolution theorem. (16%)

3. For a position vector, $F = 3e^{-2t}(i - j + 2k)$, determine the followings:

(a) velocity, (b) speed, (c) acceleration, (d) unit tangent vector, and
(e) curvature. (12%)

4. For $z = x + yi$ as complex variable, find all solutions of $e^z = 2i$ (7%)

5. Solve $\nabla^2 \theta(x, y, z) = 0$ for $0 < x < 1, 0 < y < 4, 0 < z < \infty$

with boundary conditions: $\theta(0, y, z) = \theta(1, y, z) = 0$

$$\theta(x, 0, z) = \theta(x, 4, z) = 0$$

$$\theta(x, y, 0) = f(x, y), \theta(x, y, \infty) = 0 \quad (18\%)$$

6. Find the power series solution for ODE $xy'' + (x-1)y' - y = 0$ near $x = 0$. (12%)

7. Find the general solution of ODE $x^2 y'' + xy' + (x^2 - \frac{1}{9})y = 0$ in terms of Bessel functions. (6%)

8. Find a real-valued fundamental matrix and the general solution for

the system $X' = AX$ with $A = \begin{pmatrix} 1 & 3 \\ -3 & 7 \end{pmatrix}$ (14%)