

國立高雄應用科技大學
九十七學年度碩士班招生考試
電機工程系（丙組）

准考證號碼 （考生必須填寫）

工程數學

試題 共 2 頁，第 1 頁

注意：a. 本試題共 4 題，每題 25 分，共 100 分。

b. 作答時不必抄題。

c. 考生作答前請詳閱答案卷之考生注意事項。

I. (25 分) Given the joint density function of X, Y as following

$$f_{X,Y}(x,y) = \begin{cases} \frac{(2x+y)}{a} & 0 \leq x \leq 2, 0 \leq y \leq 4 \\ 0 & \text{otherwise} \end{cases} \quad \text{Please find}$$

(a) (7 分) the value of a .

(b) (8 分) the covariance of X , and Y : $\text{Cov}(X, Y)$

(c) (10 分) the correlation coefficient of X , and Y : ρ_{XY} .

II. (25 分) Answer the following questions

(a) (7 pts) Evaluate the sum

$$\binom{13}{1} + \frac{1}{2}\binom{13}{2} + \frac{1}{3}\binom{13}{3} + \dots + \frac{1}{13}\binom{13}{13} = ? \text{ where } \binom{n}{r} = \frac{n!}{r!(n-r)!}, r \leq n$$

(b) (8 pts) Toss a fair dice three times. The first outcome is a , the second one is b , and the last one is c . Find the probability that the quadratic equation $\underline{ax^2 + bx + 2c = 0}$ has no real roots.

(c) (10 pts) Toss a fair dice twice. The first outcome is a , and the second one is

b . Define a random variable: $X = |a - b|$. Please plot the cumulative

distribution function of X .

III. (25 分) Let $T: C^2 \rightarrow C^2$ be defined by $T(a_1, a_2) = (2a_1 + ia_2, 2a_2 + ia_1)$.

$A = \begin{bmatrix} 2 & i \\ i & 2 \end{bmatrix}$ is denoted as the matrix representation of T in the standard

ordered basis of C^2 . I : be the 2x2 identity matrix.

(a) (10 分) Compute $(A - 3I)^{12}$.

(b) (15 分) For a basis β such that $[T]_\beta$ is a diagonal matrix. Find β and $[T]_\beta$.

IV. (25 分) Let $F: R^3 \rightarrow R^3$ be defined by $F(a_1, a_2, a_3) = (3a_1 + a_2, 3a_2 + 4a_3, 4a_3)$

$\alpha : \{(1,0,0), (0,1,0), (0,0,1)\}$ is denoted as the standard ordered basis.

(a) (15 分) For a basis $\beta : \{(1,0,0), (1,1,0), (4,4,1)\}$ $[x]_\alpha = (x_1, x_2, x_3)$. Please find

$[x]_\beta$ and $[T(x)]_\beta$

(b) (10 分) Use α . Find all eigenvalues and the eigenspaces of F

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